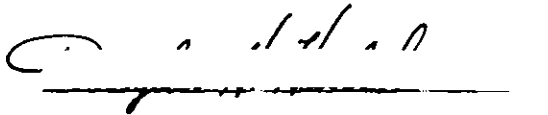


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A handwritten signature in dark ink, appearing to be "C. A. Nelson", written over a horizontal line.

3/17/65
b

A UNIT COST MODEL FOR PRODUCTION SYSTEMS

A THESIS

Presented to

The Faculty of the Graduate Division

by

Douglas Hynds Hutchinson

In Partial Fulfillment

of the Requirements for the Degree

Doctor of Philosophy

In the School of Industrial Engineering

Georgia Institute of Technology

April, 1967

A UNIT COST MODEL FOR PRODUCTION SYSTEMS

Approved:

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Date approved by Chairman: May 11, 1967

ACKNOWLEDGMENTS

The original motivation for work at this educational level was provided by Dr. J.N. Berrettoni of Western Reserve University whose counsel I value highly and whose friendship over a period of years I greatly appreciate.

My advisor during graduate study and for this dissertation was Dr. H.M. Wadsworth who has provided considerable guidance for me during the past five years.

There are numerous members of the faculty of Georgia Tech, in industrial engineering, mathematics and industrial management who have given me motivation, encouragement and, most of all, their knowledge throughout an excellent graduate program. During this time there were also numerous fellow graduate students who I will remember for their example and experience. I appreciate the opportunity that I have had to know them.

The greatest contributors to this educational goal have been my wife, Doreen, and son, David, who have experienced the trial of being the ones who have had to stand and wait.

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SUMMARY

The number of economic approaches to production system costs is large, and a great variety of types of models for a particular system makes the choice of a model a problem in itself. The present need is for one model sufficiently general to apply to most systems but with enough detail to be applicable to real processes.

This study proposes a deterministic model for a single operation that is based upon the cost accounting concepts. It contains parameters for the adjustment of unit product cost for losses due to downtime, scrapped output, purchased material rejects and worked material rejects. The single operation model is examined in terms of the sensitivity of unit cost to each process parameter.

Variations of the basic single operation model are considered. One variation is the graded output model and another is a measured work model.

The models contain parameters for overhead elements; therefore, the effect of the time value of money is not considered in the analysis of the model. The costs of interest on borrowed money, depreciation on equipment, accruals for taxes, and all other costs that ultimately are carried by good units of output are included in the overhead parameters.

Sequence of operations are then studied. The specific sequences are:

- (1) a series sequence of independent operations;
- (2) a series sequence of dependent operations;
- (3) parallel operations;
- (4) single to parallel sequences;
- (5) parallel to single sequences.

The coupling between operations is analyzed and shows that the use of in-process storages to decouple operations to provide operational independence does not make the unit costs independent. The effect of downtime upon unit cost is considered at this point for sequences of operations.

A reduced form is developed for sequences of operations, and the cost models for the reduced form are demonstrated. The application of the models to a set of examples is shown in detail in the Appendix.

Finally, the cost model is applied to a specified system of operations to determine the effect upon system unit cost of the location of storage units placed between operations within the system.

The model that is developed in this study has wide application possibilities. Its potential is based upon the use of present plant data from the cost accounting, inspection and production departments as the source of model parameters so that data is readily available and unit cost comparisons can be made in terms of complete production systems.

CHAPTER I

INTRODUCTION

Industrial engineering is the analysis and design of integrated systems composed of three factors: men, material and equipment. These three factors are the basis for the three elements of cost (material, labor and overhead) used in cost accounting. Traditionally, the systems have been manufacturing systems, but in modern times various kinds of service systems have also been considered.

Many operations research models contain product unit costs as a measure. Studies related to production systems are reported frequently in current literature, and the measure for these studies is often the total cost or unit cost of the output product. Inventories, waiting lines, forecasting, line balance, linear programming, quality control and acceptance sampling problems can be correctly developed in terms of a cost optimizing criteria provided that it is the total system that is optimized. An inventory policy optimized in terms of storage costs, interest charges and floor space may not provide a minimum unit cost for the system.

Measures of cost are generally fixed by legal and traditional accounting procedures. Laws related to taxation, labor compensation, insurance, depreciation, material

obsolescence and inventory valuation play an important role in unit cost determination. Without any doubt, unit cost is *the* measure of effectiveness in the American economic system.

The purpose of this study is to develop a unit cost model for the use of the industrial engineer. The model should be easy to understand and use; it should be designed to measure the total process costs as well as the sub-system costs; it should conform to the accounting rules and plant costing procedures; and it should be easy to apply to many different types of products and production systems. The model proposed in this study does meet these criteria.

Two kinds of cost models for production systems have been published. On one hand is the "general model" which considers the overall relationships for production systems, such as information sub-systems, and decision sub-systems. The detailed procedures for applying these models are usually omitted. On the other hand, some models are very specific and propose cost models for just one process.

The model proposed here is not too general for practical application. It is meant to be applied to any process consisting of any kind of operations or sequence of operations, and it will determine the unit cost of the output product. It is a detailed model containing real process parameters; it is deterministic, linear, and can be decomposed into material, labor and overhead elements. A reduced form is shown for the model; it is applied to a

number of different types of production systems; and the model is used to determine unit costs as a measure for best location of in-process storages.

The parameters in the model represent the expected values of process parameters measured over long periods of time in the plant or management estimates of the parameter values. The question of a test of the model is not a question of estimating parameters and determining confidence intervals for the estimates. It is rather a question of whether or not the model does determine a unit cost correctly. The test would be to use the model to determine a unit cost for a known process and to compare the value from the model to the cost accounting value.

Although the model is applied to a number of realistic examples, there is no guarantee that it can be applied to every process; but there is no guarantee that *any* known costing procedure can be applied to every conceivable process.

Summary

This chapter considers the importance of a unit cost model, the types of models that have been developed to date, and the type model that is to be developed in this research.

CHAPTER II

PRODUCTION SYSTEMS

The number of real production systems that an industrial engineer encounters in practice can be quite large, and the types and complexity of the systems are greatly varied. The operation of a process requires that problems be solved by focusing engineering efforts at one area at a time. During one period of time the problem may be measurement of the work content of certain jobs in the process. During another period the problem may be the redesign of the methods of loading or unloading a machine; during another period, a study of the use of an automatic device or new machine to increase performance at an operation; during another, the debugging and control of an operation to reduce the scrap level.

During the time he works with a process, the engineer becomes familiar with the minute details of each operation in the process. Over a period of time the process evolves and improves due to the total efforts of operators, foremen, engineers and managers. Improving a process means changing a process; e.g., bottleneck operations are adjusted to increase the capacity of the process. The causes for scrap losses are determined and eliminated. Raw material requirements are studied and specifications are changed to eliminate losses due to defective materials. Vendors are changed to

obtain better prices. Service equipment is changed and maintenance procedures are improved to reduce overhead costs.

However, these changes do not occur in a random manner. In fact, the point of this discussion is that all the problem solving efforts are managed in such a way that the cost of the unit produced is reduced by each effort applied to the process. Managers study the cost structure of a process to determine which changes can be applied most effectively. Casual observers of a process frequently note improvements that can be made but remain unchanged. This occurs because the return in unit cost improvement would not be as important to the overall process at this point as it is at other points in the process.

Thus, an accurate model of a process unit cost is important: first, because it shows clearly those operations or points in the process where improvements are most effective; and, second, the cost model contains parameters that indicate the factors in the process that can be changed. Since these factors are numerous it is inevitable that a unit cost model will contain many parameters and become awkward to manipulate. To present a model with numerous parameters is difficult unless there is some schematic arrangement of a process to describe the operation of the process and process system configuration without many, many words. Flow charts and process layouts are used to accomplish this for most processes.

A framework of process description, process configurations, process design considerations and general process

problems are now considered so that the model to be constructed can be visualized in terms of a process.

A diagram of a paper mill process is shown in Figure 1.

Wood can be reduced to fibers either by cooking wood chips with chemicals or by grinding hardwood logs into "sawdust." Pulp made by the former process is called cooked pulp and by the latter process is called mechanical pulp. A continuous digester has been proposed for this process. Prior to its proposed use, a set of four "cookers" was used. These were large pressure vessels similar to the home pressure cooker in their operation. Wood chips and chemicals were fed into the top of a cooker and the top was sealed. The batch was then cooked until the wood chips in the cooker were reduced to fibers. After further processing, the slurry (paper pulp in a water suspension) was poured in a thin layer on the moving screen of a paper machine and converted into a moving sheet of paper.

The continuous digester is a large tower into the top of which wood chips and chemicals are introduced in a continuous stream. As the mixture settles through the digester the mixture is cooked and converted into pulp. The discharge at the bottom of the digester is a continuous stream of cooked pulp ready for the next operation. The change to a continuous digester is essentially the conversion of a batch process to a continuous one.

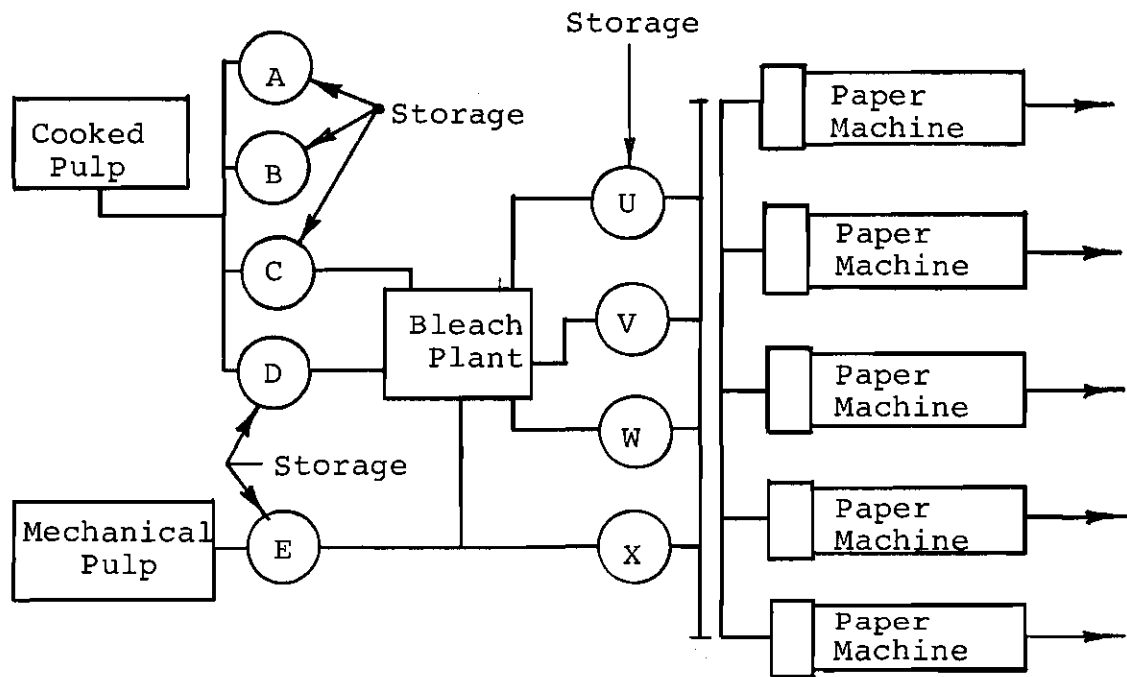


Figure 1. Operations and Storages for a Paper Mill

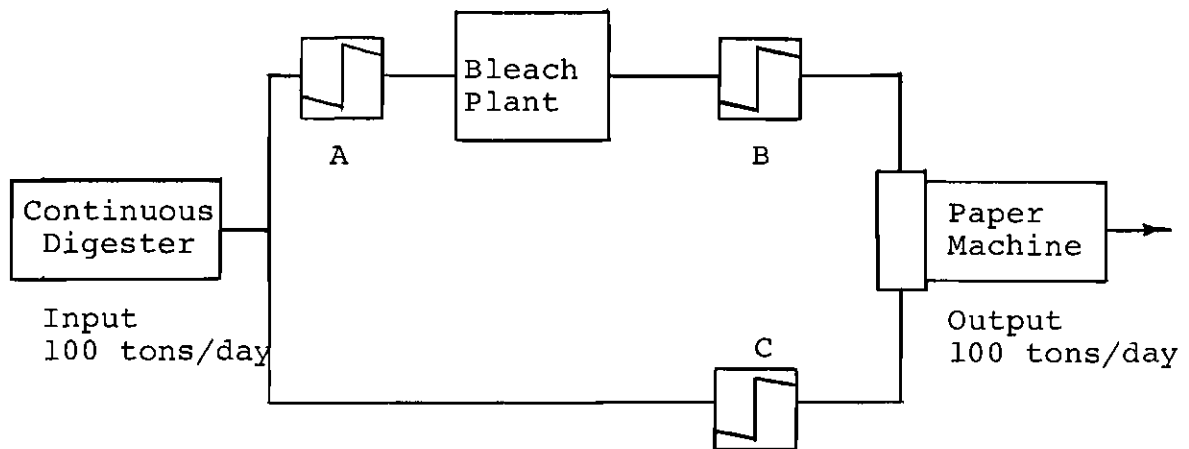


Figure 2. A Hypothetical Paper Manufacturing System

Of the many operations in the conversion of wood to paper, the only ones of interest here are cooking, bleaching, and the paper machine itself. A number of different pulp grades are produced in the process, and a batch process provides a desirable flexibility in the scheduling of pulp grades. A change to the continuous cooker for "cooked" pulp eliminates the possibility of providing this flexibility. Changeovers of the continuous process (or switching) take longer to accomplish, and best use of the operation is realized when the changeovers are minimized. Also, very large machines produce the most homogeneous and, therefore, most desirable quality pulp. This means that to the process designer one large machine is preferable to a number of smaller machines. There is, in this sense, a technical limitation on the minimum machine size with an accompanying requirement for a changeover in the process for each different pulp grade processed. The operation "switches" back and forth from grade to grade.

To show the relationship among the continuous digester, the bleach plant, and paper machine, a small segment of a hypothetical paper manufacturing system is shown in Figure 2 with these three elements and In-Process Storage (called IPS throughout this study) units. In this system the cooker switches from one pulp grade going to Tank A to another grade going to Tank C. Only the former is processed in the bleach plant, and the output of the bleach plant is stored in Tank B.

A real process would not usually mix both bleached and unbleached pulps in one paper grade. A process such as the one shown in Figure 3 shows the interrelationship among manufacturing operations. The paper machine runs continuously, drawing pulp from both Tanks B and C and using a homogeneous mixture of the two to produce the desired type of paper. This process contains all elements of the problem of this study, and similar production systems will be considered in more detail later.

A second example is shown in Figure 4 to further illustrate interrelationships among the operations in a process. Figure 4 shows the manufacture of ice cream in a dairy plant. A mixture of milk, cream, condensed milk, stabilizers, etc. is assembled, stirred and pasteurized in a 200 gallon tank. Mixes for both ice cream and sherberts are made up in the tank according to a schedule based upon demand. After pasteurization the mix is pumped through a homogenizer and then cooling coils and finally into a storage tank. The two product types are stored separately. In the final stage of production the operator in the freezer room opens a valve permitting five gallons of mix to flow into a batch freezer. The mix has a consistency similar to thick "mud" when it is discharged from the freezer into a five gallon can. Some of the cans of mix go directly to a freezing room where they are stored at 20 degrees below zero and where they harden in a few hours to a solid. The remainder of the cans are poured

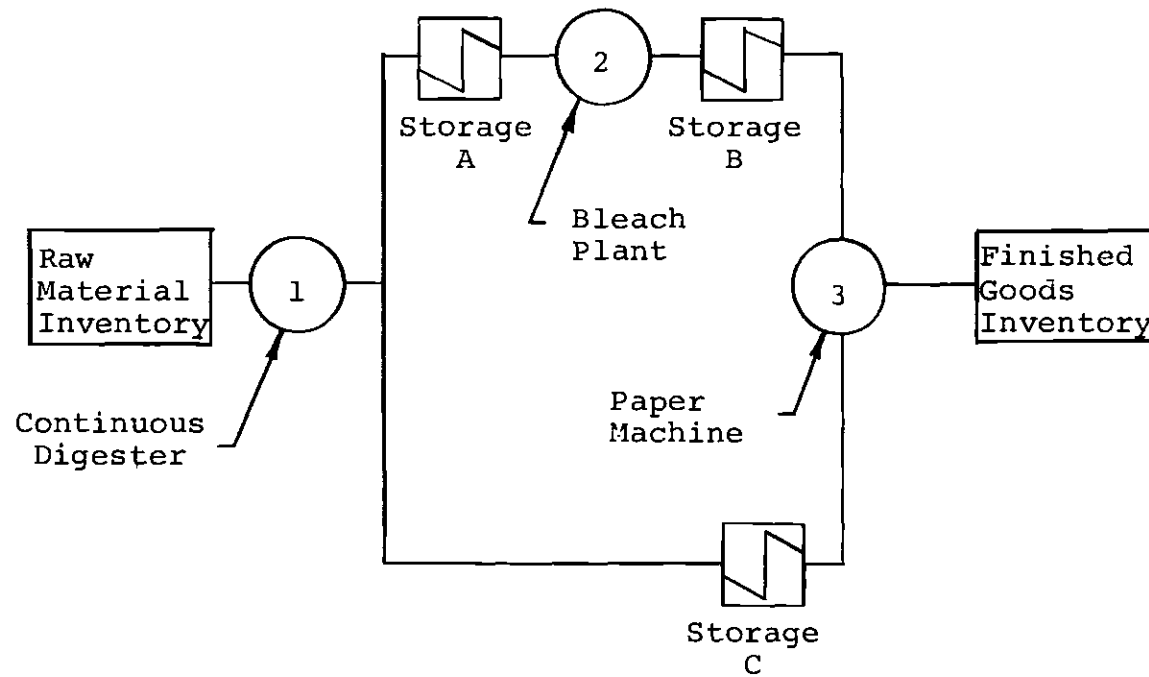


Figure 3. A Symbolic Diagram for the Hypothetical Paper Manufacturing System

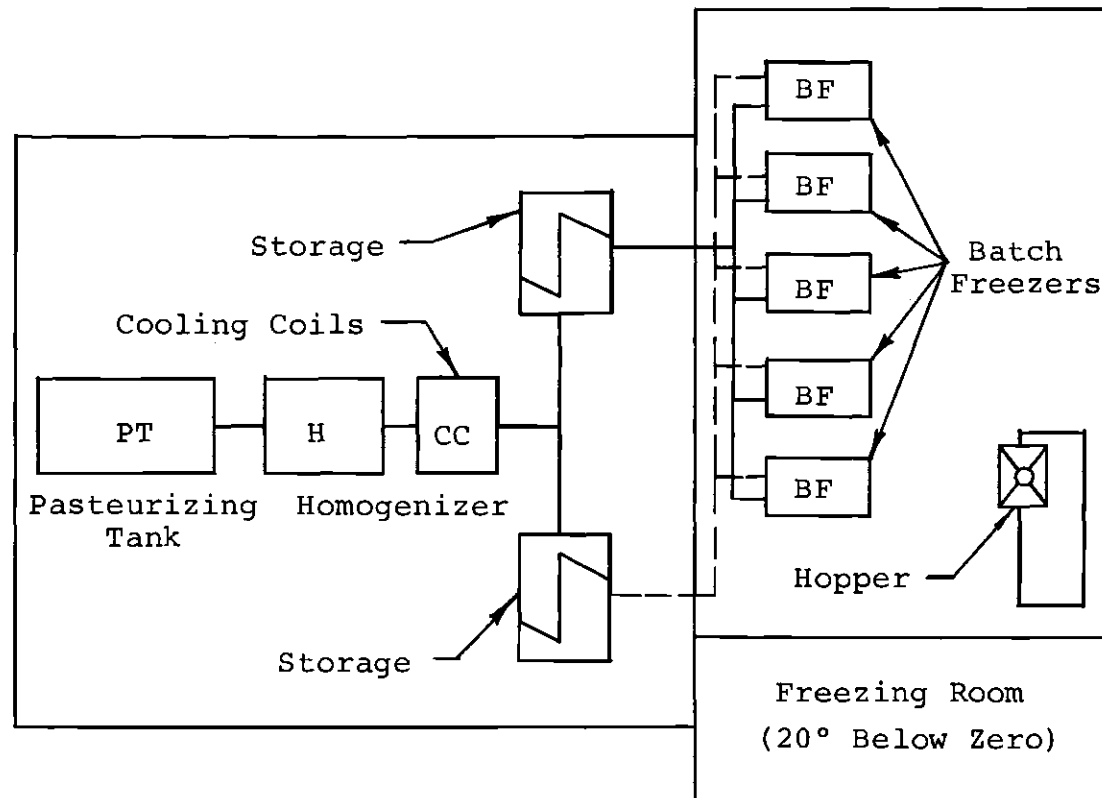


Figure 4. A Layout of an Ice Cream Manufacturing Process

into a hopper, feeding a mechanism that fills pint, quart, half gallon and gallon containers. The mix is then frozen in these containers.

In this process there is one pasteurizing tank which "switches" from batch to batch between the two type mixes. The storage tanks provide a "buffer" between the 200 gallon capacity of the pasteurizing tank and the five gallon capacity of the freezers. A symbolic diagram of this process is shown in Figure 5.

As a third example, consider a process for primary battery production. The product is called a "dry" battery in contrast to the "wet" or secondary system used for automobile batteries or railroad signal cells. Figure 6 illustrates the process. This is only a small portion of the total process, but it is a complete sub-system with the input being drawn from a raw materials inventory and the output going to a warehouse.

In this process, batches of depolarizer mix are blended in a mix room in mixing blenders which are similar to the mixers mounted on "ready-mixed" cement trucks. A number of different type mixes are produced and are scheduled for production as required by the schedule of the next operation. The blending cycle for one mix may take up to 45 minutes, and each mix is discharged into a mix buggy of the same capacity as the blending machine. The mixes are stored in the mix buggies until required in the next stage of production.

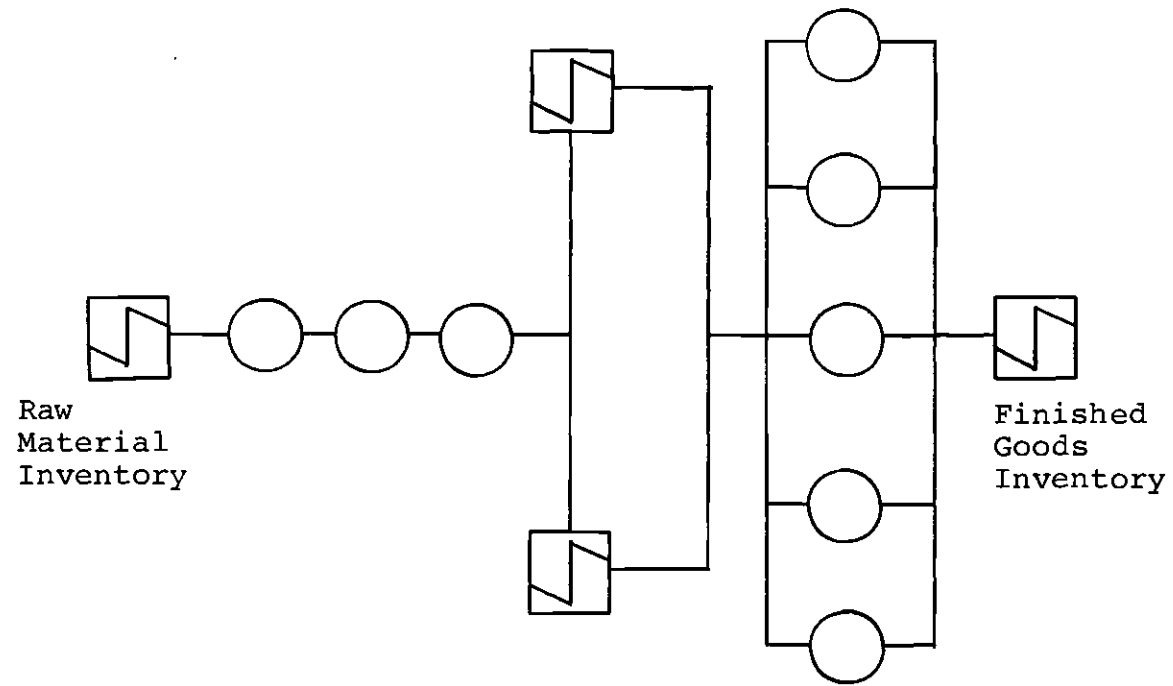


Figure 5. A Symbolic Diagram of the Ice Cream Manufacturing Process

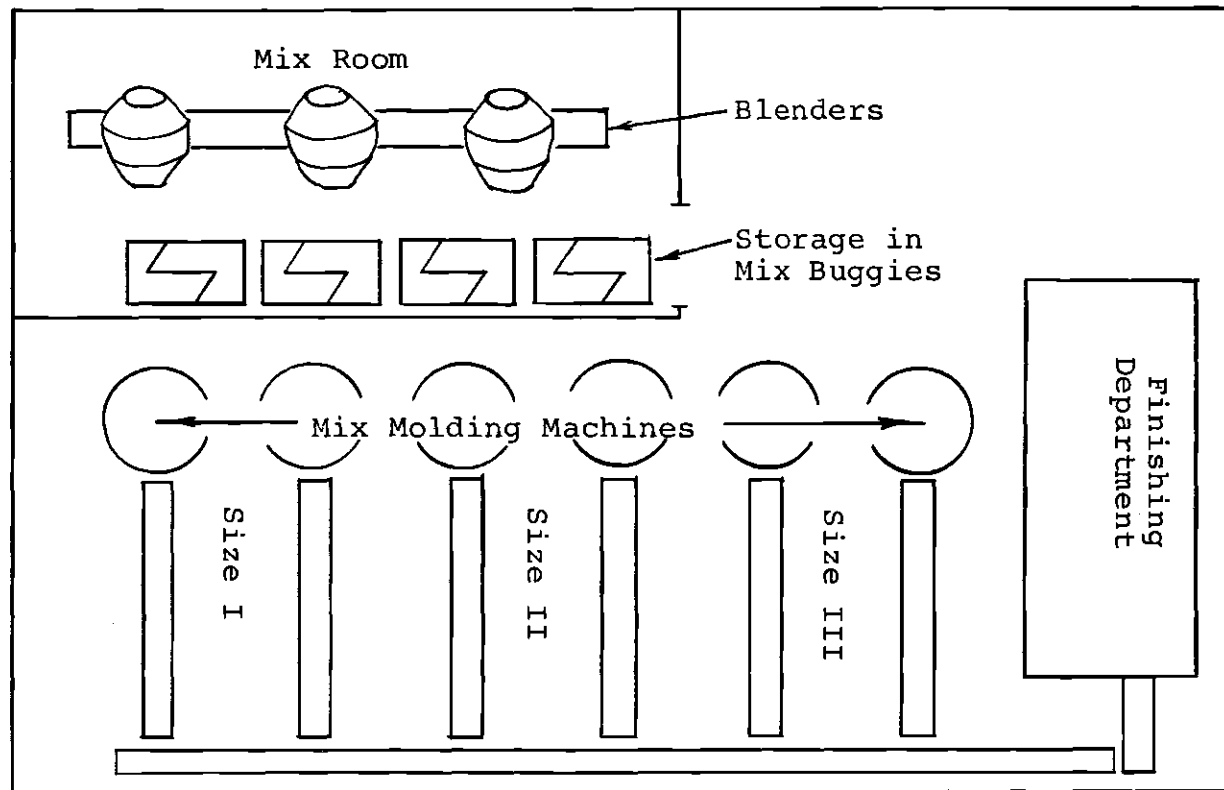


Figure 6. A Room Layout for Primary Battery Production

They are later taken to the adjoining room where the assembly lines are located. The assembly lines are divided into three different product sizes, and any size can be produced with any of the mix types. The first machine on each assembly line is a molding machine. The mix is shoveled into the hopper of the molding machine by the machine operator. The output of the molding machine is a "mix cake" which can be picked up and handled in later assembly operations but crumbles if handled roughly or dropped. The scheduled production of the assembly lines determines the number of mixes of each type to be blended each day. Type 1 mix, for example, might be eighty per cent of the output of the blenders one day and only twenty per cent the following day.

Instead of storage tanks in this process, there are mix buggies, but an identical storage function is taking place. Multiple machines are used for the production of both mix blends and mix cakes. In this case the designer of the process has chosen to use three mix blenders instead of possibly one large capacity blender. A separate mix cake machine has been installed at the end of each assembly line. The designer of the process has chosen this alternative rather than the possibility of using one high capacity mix cake machine switching from size to size with storage devices for each different size mix cake. The symbolic diagram of the process is shown in Figure 7.

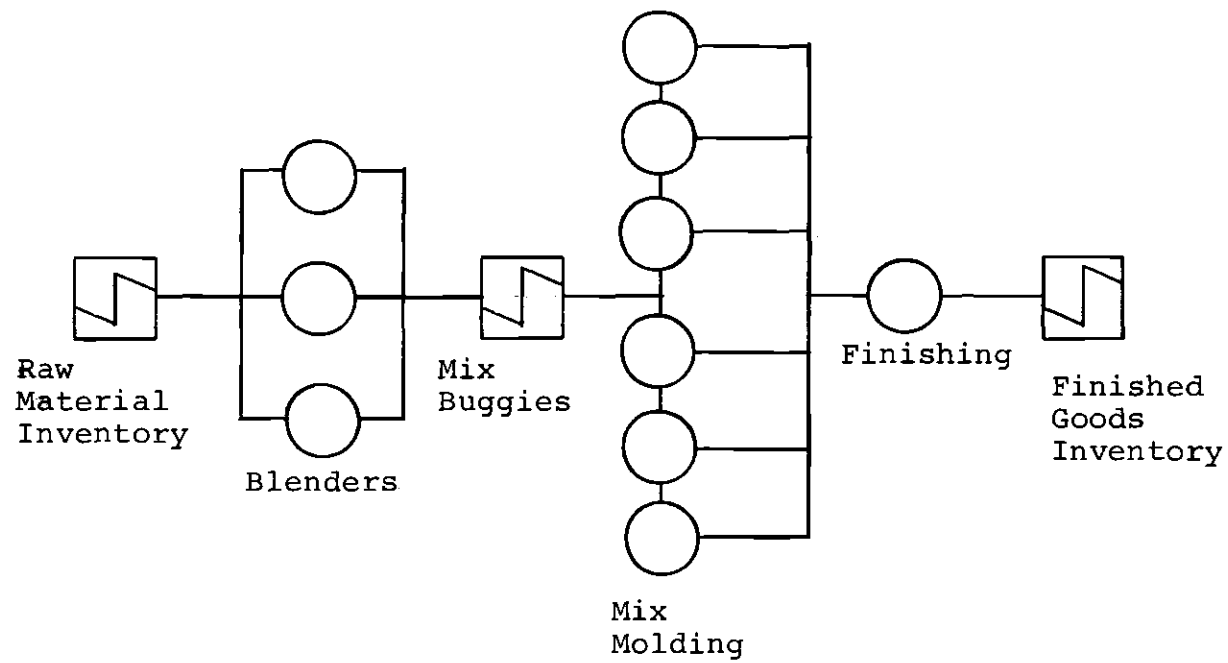


Figure 7. A Symbolic Diagram of the Primary Battery Manufacturing Process

Summary

This chapter discusses three different production system examples and shows how diagrams of the systems are drawn. The operating characteristics of these systems are shown in enough detail to acquaint the reader with the kinds of systems that are modeled in the chapters to follow.

CHAPTER III

LITERATURE SURVEY

Serial operation sequences developed first between the stations of multi-station machines. About 1914 the first real production assembly line was installed for automobile production by Henry Ford in his Highland Park plant. The key to the assembly line was high volume production of one standardized item. The total work content in the product was divided into work stations along a moving conveyor. The conveyor was a "pacing" device which maintained the rate of production.

Muther's book, *Production Line Technique* (12), was written in 1944. It was generally descriptive, like most literature of that time, but there was an occasional use of quantitative relationships to describe serial production processes.

The quantitative methods used today were not really introduced for another ten years. In 1954 a study was made by S.M. Johnson (8) of the scheduling problem; that is, the assignment of n jobs to m machines. At approximately this same time Bryton wrote his M.S. Thesis at Northwestern University (2). The next year M.E. Salveson published a paper while working at the General Electric Company (15). Both these papers were about the assembly line balancing problem.

Bryton started with a given number of work stations on a line and divided the total work content to minimize the idle time; whereas Salveson took the total work content and determined the optimum number of work stations. The measure in both cases was the amount of idle time, and both concerned themselves with a quantitative method for handling precedence relationships; i.e., the need for a certain task to be accomplished before the next could begin.

J.R. Jackson, in 1954, published a comment on the scheduling work of Johnson (5) and another in 1955 on the scheduling problem (6). In 1956 he published a computational procedure for the assembly line balancing problem (7).

These two problems, scheduling and line balancing, form the basis for the later developments in the analysis of continuous production systems. After this time, process literature develops along two paths: one the development of quantitative techniques, and the other the application of the quantitative methods to specific problems.

The survey of literature from 1960 to 1965 included numerous articles from foreign countries. The distribution is shown on the following page.

Production Process Literature by Countries
From an Initial Survey of 207 Articles

United States	124
United Kingdom	26
Russia	25
Japan	10
Germany	5
India	5
Netherlands	4
France	3
Australia	2
Sweden	1
Italy	1
Spain	1

In many cases, foreign literature is only reviewed after it is translated, and it is difficult to determine the present application of modern methods in any particular country.

Another difficulty in seeing foreign developments clearly is that much of the literature reviewed in this country is done by a journal representing some special group; i.e., The Operations Research Society of America or The Institute of Management Sciences. In Russia, the work that we would call Operations Research was called Mathematical Economics until recently changed to Economic Cybernetics. These nomenclature differences contribute further to difficulty in relating the development of a particular area of study in one country to that in another. On the other hand, there is no language barrier between the United States and the United Kingdom and developments transfer quickly between the two countries.

The literature of the Soviet Union provides a good comparison of a foreign literature to that of the United States. The picture that is generally revealed is that the Russian approach to the analysis of the production process is more applied and more detailed than in the United States and the United Kingdom. A book recently translated (17) shows an interest in the automation of processes, and the detail in which individual processes are studied is quite interesting. In our literature on production processes we have discussions of certain kinds of processing equipment and possibly an application of their use. The Russians describe complete processes in detail. Process technology in this country is generally well guarded by individual companies since new developments are provided by intensive research or possibly by outside consultants, and the companies must recover their costs in the market place.

In one area the Russian development has been similar to ours. This is Linear Programming, which had its start with an article by L.V. Kantorovich in an article published in Russia in 1939 and published in English in this country in 1960 (10).

All the reviewed publications on production processes can be put into one of 11 major categories which are listed below.

1. Assembly Line and Line Balancing
2. Scheduling and Forecasting

3. Machine Interference and Man-Machine Interference
4. Linear and Dynamic Programming
5. Process Simulation
6. Process Models
7. Costing and Process Economics
8. Queuing Applications and Waiting Lines
9. Input-Output Models
10. Process Design
11. Conveyor Theory

Topics 1, 2, 4 and 8 appeared in the literature with greatest frequency. Topics 6 and 7 are most directly related to this research. Among these references there were three of special interest.

Alcalay and Buffa (1) proposed a general model of a production system in 1963. Their purpose was to present a conceptual framework for the study of all production systems. All results are quite general, and no attempt is made to introduce data or analyze an existing system. The functions of management are defined in the viewpoint of the systems engineer, and functional definitions are given for the production sub-systems.

The mathematical statement for the production system is an input-output model. The balance equations for an entire system are presented as well as the component balance equations for man hours.

Since the purpose is the presentation of a concept, the use of example data - even for a trivial hypothetical case - was not necessary for an understanding of the paper. This is a systems engineering approach to the general model and well suited to the presentation of basic concepts.

In 1964 Reisman and Buffa (4) published *A General Model for Production and Operations Systems*. Both service and goods producing systems are included. The general model presented uses network ideas and is dynamic. Closed loop and feedback interrelationships are considered. The measure used is a financial one as would be expected for a production system. Once again, the approach is general and there is no example or application with data. The applicability of the model is discussed in terms of other models, and the authors believe that the generality of the model is demonstrated by being able to consider other models as special cases of this general input-output model. Industrial dynamic models are considered in particular and treated as special cases of this model. More than anything else, the paper stresses the generality of the model.

These two papers draw upon the most modern concepts in model development being used today and may be important for model development in the near future. They present an advanced degree of abstraction and, therefore, some work will have to be done to apply these ideas to real systems in order to prove their practical value.

In all the literature reviewed, the study most directly related to this research is a paper published in 1958 by Koenigsberg (11). It is a review of the basic problems of production and assembly lines and also considers the evaluation of the effectiveness of assembly lines. The three basic processes of analysis for production systems are listed as (1) loss transfer method; (2) the stochastic model; and (3) queuing models.

This paper is directed toward an engineering rather than purely mathematical review of the problems in the design and operation of production processes. For example, three basic problems are given as (1) the number of stages in the production line; (2) the location of IPS (in-process storages); and (3) the size of storages.

The purpose of the study is to review the methods which have been used in the analysis of production lines and arrive at an understanding of the present "state of the art." The emphasis is, however, upon the queuing approach.

The structure of production systems is first considered with some basic definitions applicable to the production process, and this section ends with the following conclusion:

Any attempt to calculate the efficiency of a production system must therefore take into account the three parameters which govern the rate of output: the two basic time distributions, cycle time \bar{r} and setting time \bar{t}' , and the distribution of stoppages.

The different models used are then considered in detail, and the notation used in the different models is summarized in table form. A specific example of an actual process is presented to clearly show the application of the models to a process.

The final paragraph in the paper is presented below because it expresses the point of view with which the proposed research - a study of the unit cost relationships for production systems - begins:

The work discussed here has all been concerned with relatively simple production line systems; more complex systems have not yet been examined in any detail by analytic methods. As mentioned earlier, modern industry has achieved high production efficiencies without having recourse to complicated mathematical analyses. So far, indeed, Operations Research workers have only made the preliminary efforts in providing some understanding of production line processes and in obtaining measures of the desirability and effectiveness of in-process storage. A more complete understanding of the interactions between stages and bunkers in simple systems will, however, aid in the design and operation of more complex production lines and should ultimately contribute to the development of automatic production systems.

Summary

This survey of the literature of production systems shows the historical development of the modern analysis of production processes. The different approaches to production system problems are considered and the literature directly related to this research are discussed.

CHAPTER IV

COSTS FOR PRODUCTION OPERATIONS

The measure that ultimately determines what is done in a production operation is the unit cost of the product. Decision makers base their actions upon this measure, and changes in equipment, labor costs and inventory policies are all related in some way to this measure.

One way to determine unit cost is to add all plant expenses in dollars for a manufacturing process for one year and divide this by the total output in units for one year to arrive at a cost per unit. For a plant with only one process, producing only one unit, in a very stable economic environment this is an actual true cost; and if no changes of any kind are anticipated for the next year this would be the measure of cost to use in the future. Many companies have done this for years, and some still do, but when changes occur it is impossible to measure the effect of a change upon the cost of a unit produced unless a detailed system for costing products exists. If new machines are purchased, labor costs, material costs and power costs change, or if more than one product is made using a number of different operations, then total cost divided by total output is a rather naive measure of unit cost.

Cost accounting techniques have been developed to estimate costs in detail and in a manner that meshes correctly with accounting and auditing procedures. Usually, this is accomplished with a system of standard costs. The elements of cost are material, labor and overhead. Material costs and overhead costs are accumulated from billing and invoices, and their control is generally external. Material costs, for example, are supplied to the purchasing agent, and it may be possible to improve these costs by bargaining with vendors. To some extent, programs of "value engineering" within the plant can alter the specification for material in a product to improve costs substantially, but once a specification is fixed the real costs are determined external to the manufacturing plant.

Standard cost systems generally concentrate upon procedures of accounting for labor. One such system is called the Normal Cost System. In this case the cost standards are based upon a combination of past experience, management objectives and allowances for process difficulties. It is considered to be realistic and is agreed upon by management and operating personnel as the unit cost value that should result under the "normal" operation of the process. It is generally established at the beginning of a year and used as a measure throughout the year. At the end of the year the normal is reviewed and revised according to experience of the past year. A separate cost is determined

for each different product. In general, a cost must be available for the coming year so that the selling price can be based upon cost values, and profit and loss in sales is based upon decision making and not waiting for the end of the year to determine "how the company came out." For those companies which submit bids and need a low bid to acquire orders, the estimated unit cost of its product is critical when there is considerable competition, and this has put an emphasis upon cost standards.

Setting normal costs begins with a sales forecast usually supplied by the sales department. It is an estimate of demand for the coming year for all existing product types. From this figure the operating level of a plant is determined. When a company has several plants the management decides where each product is to be made and what the total output for each product will be. It may decide to operate one plant for three shifts daily and another on a one shift basis. Once the operating "level" for a plant is determined and the total production schedule is known, accounting procedures are used to determine overhead costs. Overhead costs are fixed (heat, light, taxes, etc.), semi-fixed (some expenses vary with operation output but are constant over a certain range and step up to a higher value if output raises to a higher "range" of values) and, finally, variable (overhead items that can be charged to a product but which are difficult to assign as direct material costs). A purely theoretical

treatment of overhead costs would assign every dollar of expense to a product, but in practice overhead costs are assigned by some method of "prorating" costs to products. For example, heating costs can be assigned to a machine center or operation using the proportion of the total floor space in square feet assigned to the operation, and the cost of lubricating oil might be charged to a department using the proportion of the value of the machinery in the department to the total value of machinery in the plant. The accounting department also determines the price of purchased materials through the purchasing department. This is done using bills of material and material specifications once the product mix for the plant is known. Material specifications are always available, but because material price is based upon the quantities ordered this is not done until the sales forecast is prepared and purchasing quantities are established. Large companies generally purchase centrally and have large purchase contracts broken down to smaller shipments to individual plants because of the considerable economy of large purchasing. In this sense, volume plays its well understood role in material pricing and in product unit costs.

The third element of cost, labor, is determined by the industrial engineering department as a part of its function of setting and maintaining labor standards. By measuring the work content of each task in an operation the number of operators required is established and the output of an

operation is determined. If labor is actually measured, the number of standard minutes of work is known and labor requirements are based upon the measured work values. Before the normal cost is determined it is also necessary to include "normal" losses at the operations. When work is measured, output is based upon some per cent of standard performance. Operators on incentive are expected to do better than 100 per cent, and an effort level varying from 110 to 120 per cent is the usual basis for output. This is not the machine capacity, however, since real capacity is the output rate per minute for 480 minutes per shift. Downtime losses will generally reduce this and normal levels are found in practice at 70 to 90 per cent of capacity. Past experience with machine operations determines what the "normal" operating level is to be and rarely in practice does it exceed 90 per cent of true machine capacity.

There are other losses that are included in normal costs as "overusage" for both material and labor. Purchased material entering an operation is always to some extent defective. This is controlled by acceptance sampling procedures to a reasonable level. For some operations it is critical and the fraction defective can run down to 0.005 while generally it is from 1 to 3 per cent. Data from day-to-day process operations are used to determine the figure used in "normals" and the reject level is added to purchased material quantities. Also, there are scrap losses after an

operation. For small, low cost units the items actually are discarded as scrap while other type items are repaired and "made good." In either case, the loss of units which have been processed always exists, and this value is included in the normal cost. Labor costs above normal occur through absenteeism, learning, and because operators on piece work fail to make the guaranteed minimum. These labor costs are also added as percentages to labor costs.

This discussion of the factors in a Normal Cost are well understood by both the practicing industrial engineer and management personnel who are accustomed to thinking in terms of unit costs in the decision making process. Principally for this reason, the models for manufacturing processes will use unit cost as the measure. All the elements of normal cost are included in the models for unit cost, although not in the form used by any particular company.

Summary

The methods presently used in production system costing are discussed in this chapter. To the industrial engineer who is not accustomed to product cost determinations it is important to understand how changes and improvements are measured by management.

CHAPTER V

THE SINGLE OPERATION MODEL

The Daywork Model

All operations can differ in the type output that is developed. On the one hand are discrete outputs where individual units can be inspected before going to storage or the next operation; and on the other, continuous or flow outputs where the output is a homogeneous mass, such as a liquid, bulk solid or gas, and passes to storage or the next operation intact. The continuous output may be separated into two or more grades, but does not have any physical separation into units. If, for example, the output is 100 gallons, we do not think of each gallon being inspected as a discrete unit. The result of inspection may be 80 gallons of first grade and 20 gallons of second grade product.

Inputs to operations can differ in the same way and the input and output can be of different types. The inputs to a canning machine might be 100 gallons of solution and 100 containers. The output would be 100 discrete units or one gallon cans of solution. The input and output units for any operation indicate the characteristics of the inputs and outputs. Changes in type present no special problem for cost models, however.

The diagram for a single operation with discrete input and output is shown as Figure 8. Notation for the single operation is as follows:

- K_i = The capacity for operation i . Capacity is the rate of *output* per minute times total time per shift. It is the theoretical maximum output.
- b_i = A capacity factor which is applied to the operation factor K_i so that $K_i b_i$ represents the actual expected output from an operation. It is a dimensionless number.
- c_{i-1} = Unit cost of worked material from the previous operation. This is material that has been processed through operations in the same plant and same cost system. This cost differs from purchased material cost since it carries elements of material, labor and overhead.
- r_i = Reject proportion or fraction applied to incoming material.
- m_i = Purchased material cost per unit.
- t_i = A reject factor for purchased material representing the fraction of the purchased material rejected at the operation.

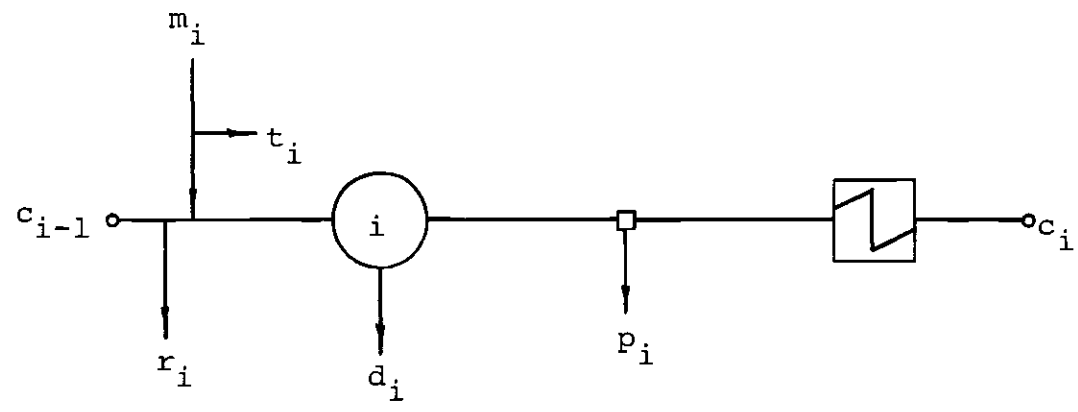


Figure 8. Diagram for the Single Operation Model

G_i = Guaranteed day rate cost that applies to operations on daywork. Piece work operations also carry a guarantee, but a separate model is developed for measured labor operations.

k_i = Overhead costs per unit. This is the variable portion of overhead cost that can be applied to each unit.

f_i = Fixed portion of overhead cost expressed in dollars of total cost.

s_i = Semifixed portion of overhead cost expressed in total dollars assigned to operation i .

d_i = Downtime loss shown in the model as a proportional adjustment to K_i .

p_i = Fraction loss in finished units. This is the scrap rate.

This notation and all additional symbols used in this paper are shown in the Glossary of Symbols in the Appendix.

The model for a single operation is:

$$c_i = \frac{A_i X_i + B_i}{X_{gi}} \quad (1)$$

$$\text{where } A_i = c_{i-1}(1+r_i) + m_i(1+t_i) + k_i$$

$$X_i = K_i b_i (1-d_i)$$

$$B_i = G_i + f_i + s_i$$

$$X_{gi} = K_i b_i (1-d_i)(1-p_i)$$

The unit cost in dollars per unit measured after operation i is c_i .

The r_i proportion applied to the input from a previous operation measures the loss at an inspection operation that is directly related to an operation in the same plant. The reject value attached to m_i reflects a loss related to material purchased from an outside vendor or another plant in the same company. In the unit cost model these two losses are shown by an "expected" quality measure.

As an example of the use of this model, assume the values given in Table 1 for a single operation. For one operation the subscript $i = 1$ and subscript $i - 1 = 0$. Then;

$$\begin{aligned} A_1 &= 1.00(1+0.02) + 2.00 + 0.03 = \$3.05/\text{unit} \\ X_1 &= 100(1)(1-0.08) = 92 \text{ units} \\ B_1 &= 24.00 + 20.00 + 20.00 = \$64.00 \\ X_{g1} &= 100(1)(1-0.08)(1-0.02) = 90 \text{ units} \end{aligned}$$

and

$$c_i = \frac{A_1 X_1 + B_1}{X_{g1}} = \frac{3.05(92) + 64}{90} = \$3.83/\text{unit}$$

In these calculations both X and X_g are rounded to integral values.

The single operation unit cost model can be manipulated in a number of ways to determine the effect upon unit

Table 1. Parameter Values for the Single Operation Model

System Parameter	Symbol	Units	Operation 1
Operation Capacity	K_1	units/shift	100
Capacity Factor	b_1	-	1.00
Scrap Loss Factor	p_1	-	0.02
Downtime Factor	d_1	-	0.08
Worked Item	c_0	\$/unit	1.00
Reject Factor	r_1	-	0.02
Purchased Item	m_1	\$/unit	2.00
Reject Factor	t_1	-	0
Total Daywork Labor	G_1	\$/shift	24.00
Total Variable Overhead	k_1	\$/unit	0.03
Total Fixed Overhead	f_1	\$/shift	20.00
Total Semifixed Overhead	s_1	\$/shift	20.00

cost of decisions made about the process. The most obvious consideration is the real meaning of the normal cost. When process parameters are determined by decisions about expected output values and expected scrap and reject values, the resulting normal cost can be measured against a theoretical "perfect" value. In other words, the minimum obtainable unit cost is available when the model is evaluated with no losses. For this example the "perfect" unit cost is;

$$c_1 = \frac{A_1 X_1 + B_1}{X_{g1}}$$

where $A_1 = 1.00(1+0) + 2.00(1+0) + 0.03 = \$3.03/\text{unit}$

$X_1 = 100(1)(1-0) = 100 \text{ units}$

$B_1 = 24.00 + 20.00 + 20.00 = \64.00

$X_{g1} = 100(1)(1-0)(1-0) = 100 \text{ units}$

and

$$c_1 = \frac{3.03(100) + 64.00}{100} = 3.03 + 0.64 = \$3.67/\text{unit}$$

The operation of this production process begins with an input from a previous operation within the same plant. A cost for material from a previous operation, c_{i-1} , entering operation i exists when the output of $i-1$ goes directly from $i-1$ to i . It may pass from one station to the next station

within one department or it may pass from one department to another. In-process inventories may be involved, but inventories from which material has to be requisitioned are not. Requisitioned materials from inventory would be treated as purchased material. Work from a previous operation carries a reject allowance, r_i , whenever an inspection takes place between the operations. If there is no inspection, r_i is zero; but this does not mean the worked material is 100 per cent good. If there is no inspection between operations, defective units will appear in the inspection after the operation as p_i or at a later stage in the process. The reject allowance is a parameter in the model representing the quality level of material coming from a previous operation. There may be several input materials from previous operations. Figure 8 shows the general model, but Equation (1) has only one input representing all possible previous operation inputs.

A second source of input material is purchased material with a unit cost, m_i , and with a defective fraction denoted t_i . For each purchased material a t_i value can be specified and put into the unit cost model.

The model assumes a capacity, K_i , and capacity factor, b_i , for the process, but the input to the process is adjusted by a downtime factor. The number of units of input to a machine is not $K_i b_i$ but, rather, $K_i b_i (1 - d_i)$ where d_i is a proportional downtime adjustment. A machine has both running

time and downtime. Should the machine fail in some manner while running, it can produce defective units. While producing defects the machine is running and material is supplied to the machine. When it is down, material is not consumed; therefore, the input to the machine in units is $K_i b_i (1-d_i)$, and the output in good units plus defective units is $K_i b_i (1-d_i)$. Suppose a proportion, p_i , of the output units are defective, then $K_i b_i (1-d_i) (1-p_i)$ good units are produced, and this number is the denominator for the unit cost calculation.

The downtime fraction, d_i , can be more precisely defined so that for extensions of the single operation model there will be no complication. Consider a shift of 480 working minutes to consist of 480 discrete one minute intervals. If the operation is down during any portion of a one minute interval, that interval is "down." The set of all "down" minutes divided by the number of one minute intervals per shift is the value d_i .

The fraction defective, p_i , can also be more precisely defined. If the output of a machine consists of discrete units of product, p_i is the number of defective units divided by the total number of output units. The total number of output units is separated into proportion p_i defective and proportion $(1-p_i)$ good. For output products that are continuous, proportions p_i and $(1-p_i)$ are not necessarily based upon integer values.

The treatment of labor costs in the model assume a fixed hourly rate for eight hours each day. The operating procedures for the process do not affect the total labor requirement. If a group of operators are required, the labor cost, G_i , at operation i is eight times the *sum* of the hourly costs.

Overhead costs in the model are shown as fixed, f_i , semi-fixed, s_i , and variable, k_i . Of the three cost elements, overhead is handled with greatest variety from company to company. Some firms make no attempt to separate fixed and semi-fixed costs. At times, variable overhead costs are handled as fixed costs because of the difficulty in assigning variable expenses to a specific product. All three types of overhead are shown in this unit cost model to make it clear how each affects unit cost.

Material Balance

For some processes the inputs to an operation are discrete units and one input unit is converted to one unit of output. But there are different possible input-output relationships and they are described here by the term "material balance." In all cases where one or more input units result in an output of *one* unit, the material balance is called "normal."

One other type balance is considered: this is when the output in units is the sum of the input units or related directly to the number of input units. For example,

an input of 30 gallons from operation 1 and 30 gallons from operation 2 result in an output of 60 gallons from operation 3. This is called an "additive" material balance.

Numerous examples are possible of both types of material balance. In the normal case consider the assembly of an automobile when one engine, one body, one chassis and four tires are put together and one automobile is the output from the operation. On the other hand, a batch of depolarizer mix for the manufacture of dry batteries is made from X_1 pounds of ingredient 1, X_2 pounds of ingredient 2, X_3 pounds of ingredient 3, X_4 pounds of ingredient 4 and X_5 pounds of ingredient 5. If 100 pounds of each ingredient go into a mix the result is a 500 pound batch.

In costing the final product the "usage" per 1000 finished units is determined. Suppose this is 262 pounds. If the unit mix cost is based upon 1000 pound mixes, the number of units of mix per 1000 finished units is 0.262; or if the units for mix is 100 pounds, then 2.62 units of mix are required per 1000 finished units of final product.

The question of material balance must be clearly understood to utilize a unit cost model. Throughout this study normal one-to-one material balances are used to simplify the development of models.

To illustrate changes in material balance, consider a normal balance in the following equation:

$$c_4 = \frac{A_4 X_4 + B_4}{X_{g4}}$$

where

$$A_4 = c_2(1+r_4) + c_3(1+r_4) + m_{41}(1+t_{41}) + m_{42}(1+t_{42}) + k_4$$

$$X_4 = K_4 b_4 (1-d_4)$$

$$B_4 = G_4 + f_4 + s_4$$

$$X_{g4} = K_4 b_4 (1-d_4) (1-p_4)$$

In this case, worked material comes from operations 2 and 3 and purchased materials of two different types are added at operation 4. Only one unit of each type material is required. Now consider this equation with a normal material balance but requiring two units of worked material 2, two units of worked material 3, one units of purchased material 41 and three units of purchased material 42 for each output unit from operation 4. Then A_4 becomes;

$$A_4 = 2c_2(1+r_4) + 2c_3(1+r_4) + m_{41}(1+t_{41}) + 3m_{42}(1+t_{42}) + k_4$$

This does not present any special problem in the development of the various necessary models.

Graded Output Model

A variation of the single operation model is the graded output model. It applies to both discrete and

continuous products, but more so to the continuous products. An application to a discrete product is the production of hosiery in the knitting mill. Such a process is shown as Figure 9.

In the first two operations a stocking is knitted and the toe sewed closed. Consider operation 2 as a single operation with one input and three outputs. Up to the final inspection operation there is one product and the final inspector sorts the product into grades as well as rejecting a fraction, p_2 , as waste. The second grade products are repairable. After the subsequent repair operation an inspector grades the stockings into two grades.

Continuous products such as gases, liquids, streams of bulk products and slurries are frequently graded. In this case there is no inspection before grading and the fraction defective is the "lowest grade." With continuous products it is not possible to physically separate the yield of the process into good and bad units because it is homogeneous, but using devices such as filters a grading can be accomplished. The diagram of a process with a continuous product is shown as Figure 10. Grade 3 is the lowest quality grade and might be considered waste or scrap. In terms of the model it would make no difference whether it was called p_1 or grade 3.

The basic formulation of unit cost is demonstrated in the calculation of unit cost for graded output products.

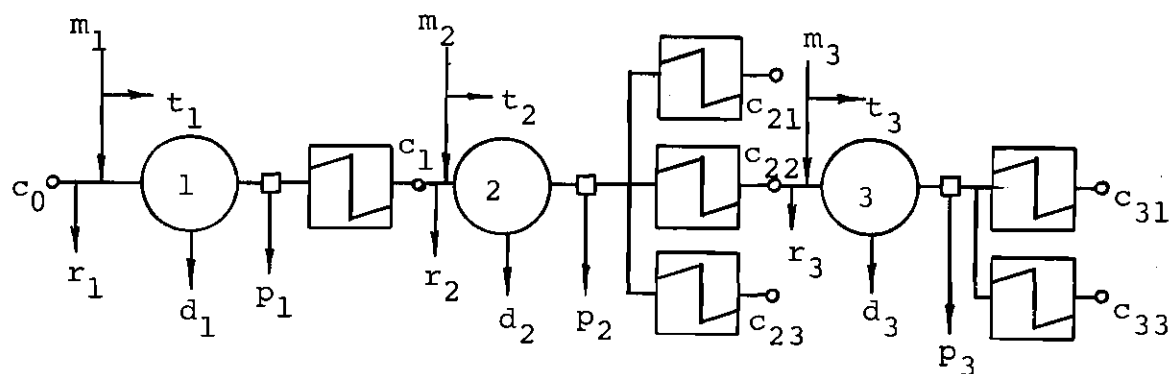


Figure 9. Diagram for the Graded Output Model.
Discrete Product Case

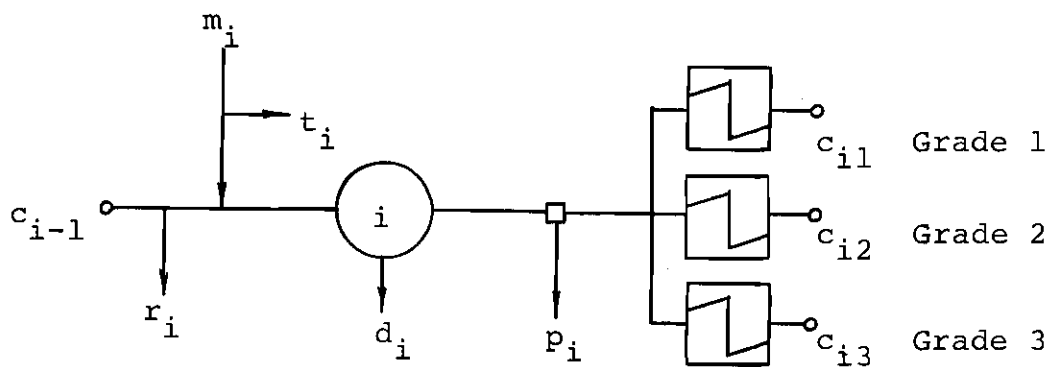


Figure 10. Diagram for the Graded Output Model
Continuous Product Case

The input to the operation in Figure 10 in number of units is $K_i b_i (1-d_i)$. The output which is divided into grades 1 and 2 is $K_i b_i (1-d_i)(1-p_i)$ units. The total input cost divided by total output units is the unit cost of output. How the output is graded has no effect upon unit cost, although it would affect profit when the output is later sold. The cost equation is;

$$c_i = \frac{A_i X_i + B_i}{X_{gi}} \quad (2)$$

$$\text{where } A_i = c_{i-1}(1+r_i) + m_i(1+t_i) + k_i$$

$$X_i = K_i b_i (1-d_i)$$

$$B_i = G_i + f_i + s_i$$

$$X_{gi} = K_i b_i (1-d_i)(1-p_i)$$

Measured Work Model

Another variation of the single operation model presents a more detailed model for the labor elements of unit cost. The total labor work content of an operation should be measured to obtain control over labor requirements and labor cost. Day work payment means that a fixed rate is paid per hour to the operator. The payment may or may not be based upon the work content of the operator's task. Straight hourly rates are handled in the unit cost model by the factor G where G is the product of dollars per hour and hours per shift. When a crew of two or more operators is

required the value G is the sum of the eight hour payment for all the operators. It makes no difference in this study how the hourly rate is established.

When the output of an operation is operator controlled, the work content of the job determines the unit cost in the manner discussed below.

Normal time is the time required by an operator to produce a unit of work at a "100 per cent" effort level. Normal time usually is determined in minutes per unit. When allowances are added to normal time it is converted to standard time and measured in the same units. The number of standard minutes of work required to produce one unit is the basis of payment in measured work systems. The operator is "paid" in minutes of work, and the standard minutes of work are converted to dollars by a base rate for wage payment. Also, standard minutes determine operation capacity.

In normal cost systems payment for labor is based upon an *expected* effort level rather than 100 per cent. Past experience generally determines the expected percentage or performance level that is to be used in cost determination. For this study 120 per cent will be used.

Now suppose that the standard time for an operation is 6.40 minutes per unit. An operator working at 120 per cent would only require 5.34 minutes to perform the work per unit, and operation capacity is;

$$K_i = (480 \cdot \frac{1}{sm_i}) \text{ divided by } Ef_i$$

$$= \frac{480 \cdot Ef_i}{sm_i}$$

where K_i = the output capacity of the operation for an eight-hour or 480-minute shift;

sm_i = standard minutes per unit for operation i ;

Ef_i = performance level expected for operation i .

For example:

$$K_i = \frac{480 \text{ min.}}{\text{shift}} \times 1.20 \times \frac{\text{units}}{6.40 \text{ min.}} = 90 \text{ units/shift}$$

For the labor cost in dollars per shift we use the base rate in dollars per hour times the eight hours per shift to determine the labor cost, G_i . Since the dollar earnings of the operator depend upon his performance level this quantity should be multiplied by Ef_i . Then;

$$G_i = 8 \cdot br_i \cdot Ef_i$$

where G_i = dollar earnings paid to the operator;

br_i = the base rate for operation i ;

Ef_i = the performance level expected at operation i .

To convert this value to a unit cost, the output in units is determined again as;

$$X_{gi} = K_i (1-d_i) (1-p_i)$$

and since $K_i = 480 \cdot Ef_i / sm_i$, we have a labor unit cost equal to;

$$c_i^L = \frac{G_i}{X_{gi}}$$

where c_i^L = the labor element of unit cost;

$$G_i = 8 \cdot br_i \cdot Ef_i$$

$$X_{gi} = \frac{480Ef_i}{sm_i} (1-d_i) (1-p_i)$$

This shows that the labor cost for a measured work system is not affected by the effort level of the operator. This also implies that the labor element for this measured work model is for a "straight piece work" wage plan. The complete cost model is altered because capacity is determined by the operator.

The model becomes;

$$c_i = \frac{A_i X_i + B_i}{X_{gi}} \quad (3)$$

where $A_i = c_{i-1}(1+r_i) + m_i(1+t_i) + k_i$

$$X_i = \frac{480 \cdot E f_i}{s m_i} (1-d_i)$$

$$B_i = 8 \cdot b r_i \cdot E f_i + f_i + b_i$$

$$X_{gi} = \frac{480 \cdot E f_i}{s m_i} (1-d_i) (1-p_i)$$

In the previous models the output in units was $K_i b_i$ so that operator controlled operations actually replace $K_i b_i$ by;

$$K_i b_i = \frac{480 \cdot E f_i}{s m_i}$$

To work with this model, several new parameters are used. The symbol for the parameters in this model as well as the previously defined symbols are found in the GLOSSARY OF SYMBOLS.

Summary

This chapter presents the formulation of the model and is the central point in the study. The model, the parameters, the reasoning underlying the modeling procedure and the meaning of the principal symbols are all here discussed. Examples of variations upon the basic model are presented and discussed.

CHAPTER VI

THE GENERAL SINGLE OPERATION MODEL

The single operation model represents the least complex manufacturing operation. It applies to both discrete and continuous products, has been discussed for graded output products and modified for measured work applications. Rather than consider other special models, a general single operation model is now developed to describe any manufacturing process with material, labor and overhead elements.

If some costs are unit costs when applied to the product and others are total costs, the unit product cost in its most general form is written;

$$y = A + \frac{B}{X_g}$$

where y = unit cost;

A = material, labor and overhead elements
applied as unit costs;

B = material, labor and overhead elements
applied as total dollars;

X_g = output of the process in good units.

In other words, some costs are applied to a process in unit cost form, and the remainder need to be converted to unit cost form by dividing by the output in units. Inputs can be either in dollars or dollars per unit, but outputs must be in dollars per unit.

No consideration is given in this expression to any process losses. By accumulating all costs per unit as A and all total costs over a specified time interval as B, the overall unit cost is found if the total output in units during the time interval is known.

As an example, suppose that for a year the sum of unit material costs is \$2.00 per unit of product and the sum of variable overhead is \$1.10 per unit; also, the total labor cost for the year is \$18,000.00 and the total overhead costs for heat, power, rent, taxes, interest, depreciation, etc. is \$100,000.00. Assume that 105,000 units of product are made. Unit cost would be;

$$y = \$3.10 + \frac{\$118,000}{105,000 \text{ units}}$$

and $y = \$4.22$ per unit; then, since y represents unit cost;

$$y = \frac{\text{Input Cost}}{\text{Output in Good Units}}$$

Input costs are charged to the operation in two ways: one as unit cost of inputs, and the other as total dollars of inputs. We called the former A and latter B. The input quantity is called X and the output in good units X_g . Then a second form for unit cost is;

$$y = \frac{AX + B}{X_g}$$

This is the form used for the unit cost model. The input quantity X can be written $Kb(1-d)$ to include the downtime loss. By a consideration of scrap losses as well as downtime, the output, X_g , in good units can be written as;

$$X_g = Kb(1-d)(1-p)$$

The factor A accounts for the unit cost of purchased material, worked material and variable overhead, or;

$$A = y_{i-1}(1+r_i) + m_i(1+t_i) + k_i$$

Finally, B includes total dollar costs for labor and expense, so $B = G_i + f_i + s_i$. Substitution of these factors into the general form and writing c_i rather than y gives the detailed unit cost model;

$$c_i = \frac{[c_{i-1}(1+r_i) + m_i(1+t_i) + k_i][K_i b_i(1-d_i)]}{K_i b_i(1-d_i)(1-p_i)} \quad (4)$$

$$+ \frac{G_i + f_i + s_i}{K_i b_i(1-d_i)(1-p_i)}$$

Some of the more complicated systems models that are discussed later are approached using the general form;

$$y = \frac{AX + B}{x_g}$$

The denominator of the unit cost expression is the output in good units. Output is determined by the capacity of the operation where capacity is a theoretical ideal value expressed as K. This capacity is the rate at which units are produced with no loss through scrap or sampling and no downtime at the operation for adjustments or maintenance. When the operation is machine paced, the machine speed determines output. When the operation is operator paced, the effort level of the operator determines output. This is why operator-paced operations must be measured and also why a "standard" effort level (we used 120%) is assumed for unit cost calculations.

The operation capacity is reduced by an amount "d" which is the fraction of the capacity lost through stops or downtime, or through running losses that occur as the machine is running (for example, when an indexing feed

mechanism fails to pick up a unit and that machine position runs empty for a cycle), or because of a gradual decrease in efficiency (for example, a cleaning bath becomes diluted and the speed of the machine is decreased to provide a constant exposure of parts to the cleaning solution). The loss in capacity is dK . The number of units that come out of the operation is $K(1-d) = K - dK$. Since inspection and sampling after the operation cause an additional loss of a fraction, p , the total output in good units is $K(1-d)(1-p)$. Both d and p could be sub-divided into $d_0 + d_1 + d_2 + \dots$ and $p_0 + p_1 + p_2 + \dots$ as, for example, p might be expressed as;

p_0 = fraction of loss due to scratched case;

p_1 = fraction of loss due to defective amplifier;

p_2 = fraction of loss due to cold solder
connections;

p_3 = fraction of output taken for sampling;

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so a more general result would be to write the denominator as;

$$K_i (1 - \sum_j d_{ij}) (1 - \sum_0 p_{i0})$$

There is also the fact that both d and p may be functions of time. An example of this for p would be die wear. Suppose as a die wears the dimensions of the product increase. In the case where the die is made to a specification the increase in die wear in time results in a gradual increase in the per cent of the output outside the specification limits, and p is then a function of the number of units produced, or time for a constant output rate. We would have to know the function in order to determine the change in cost. Most production situations consider either that the change is linear or else specify the end point. For example, production operators will run a machine for 30,000 cycles after a die change. In the cleaning bath example, it might be known that concentration decreases exponentially in time. This relation could be used to determine machine speed adjustments and the change in unit cost in order to determine the best time to adjust the cleaning solution concentration. These functional considerations would mean that the denominator for unit cost would become;

$$K[1-f_d(t)][1-f_p(t)]$$

For this paper this would be the most general expression of process output.

Total cost is $AX + B$, and for the model presented it is;

$$K_i(1-d_i)[c_{i-1}(1+r_i) + m_i(1+t_i) + k_i] + G_i + f_i + s_i$$

For total cost the capacity value would be adjusted by $f_d(t)$ as in the denominator, so this term becomes $K_i[1-f_d(t)]$.

There may be several different worked and purchased materials so that both c_{i-1} and m_i would be sums of the individual unit costs multiplied by reject or overusage factors as;

$$\sum_j c_{i-1,j}(1+r_{ij}) + \sum_k m_{ik}(1+t_{ik})$$

Labor and overhead factors are treated as totals even though it is clear that both are sums of the individual labor elements and the individual overhead elements.

Even though the theoretical output from the process is K , the output may be intentionally reduced to balance operations, prevent unnecessary machine wear or meet an output decision by management. To preserve the *true* capacity meaning of K , a factor, b , is used to represent adjustments to the *true* K value. This changes K to Kb in the model and b is called a Capacity Factor. If the process or machine operates at maximum capacity, $b = 1.00$.

The numerator then becomes;

$$A_i X_i + B_i$$

$$\text{where } A_i = \sum_j c_{i-1,j} (1+r_{ij}) + \sum_k m_{ik} (1+t_{ik}) + k_i$$

$$X_i = K_i b_i [1-f_d(t)]$$

$$B_i = G_i + f_i + s_i$$

and the general model for unit cost for a single operation is;

$$c_i = \frac{A_i X_i + B_i}{X_{gi}}$$

$$\text{where } A_i = \sum_j c_{i-1,j} (1+r_{ij}) + \sum_k m_{ik} (1+t_{ik}) + k_i$$

$$X_i = K_i b_i [1-f_d(t)]$$

$$B_i = G_i + f_i + s_i$$

$$X_{gi} = K_i b_i [1-f_d(t)] [1-f_p(t)]$$

For the measured work case, K_i becomes a function of the standard minutes of work and the effort level, so that;

$$K_i b_i = \frac{480 \cdot E f_i}{sm_i}$$

and the labor earnings depend upon the base rate and effort level, or;

$$G_i = (8 \cdot) (br_i) (E f_i)$$

It should be noted here that should a daywork operation be transformed into an equivalent measured work model using

$$K_i b_i = \frac{480 \cdot E f_i}{s m_i}$$

the performance level would be 100 per cent or 1.00, so that $K_i b_i = 480/s m_i$.

The general equation is;

$$c_i = \frac{A_i X_i + B_i}{X_{gi}}$$

$$\text{where } A_i = \sum_j c_{i-1,j} (1+r_{ij}) + \sum_k m_{ik} (1+t_{ik}) + k_i$$

$$X_i = \frac{480 \cdot E f_i}{s m_i} [1-f_d(t)]$$

$$B_i = (8 \cdot E f_i \cdot b r_i) + f_i + s_i$$

$$X_{gi} = \frac{480}{s m_i} [1-f_d(t)] [1-f_p(t)]$$

The general model can be separated into the individual elements of material, labor and overhead. This will aid in the later analysis of the general model.

For material we have;

$$c_i^M = \frac{A_i^M X_i}{X_{gi}} \quad (5)$$

where c_i^M = the material element of unit cost

$$A_i^M = \sum_j c_{i-1,j} (1+r_{ij}) + \sum_k m_{ik} (1+t_{ik})$$

$$X_i = K_i b_i [1-f_d(t)]$$

$$X_{gi} = K_i b_i [1-f_d(t)] [1-f_p(t)]$$

and then labor;

$$c_i^L = \frac{G_i}{X_{gi}} \quad (6)$$

where c_i^L = the labor element of unit cost

$$G_i = 8 \cdot br_i \cdot Ef_i$$

$$X_{gi} = \frac{480 \cdot Ef_i}{sm_i} [1-f_d(t)] [1-f_p(t)]$$

and, finally, for overhead;

$$c_i^0 = \frac{A_i^0 X_i + B_i^0}{X_{gi}} \quad (7)$$

where c_i^0 = the overhead element of unit cost

$$A_i^0 = k_i$$

$$X_i = K_i b_i [1-f_d(t)]$$

$$B_i^0 = f_i + s_i$$

$$X_{gi} = K_i b_i [1-f_d(t)] [1-f_p(t)]$$

One final comment is needed for the general unit cost model. There are certain kinds of problems that affect overhead values. The one that frequently presents itself is the inventory situation where the solution to a problem determines the "best" inventory levels and, hence, storage space costs, inventory taxes and interest costs. These costs are prorated back to each operation or machine center in the process, and it would have been possible to provide for these kind of adjustments in the fixed and semi-fixed overhead factors f_i and s_i . The reason it has not been done here is that many different special problems can affect overhead costs in different ways, and the user of the model can later make the necessary adjustments to suit his purpose.

Summary

The single operation model is explained in detail in this chapter. Each parameter is considered. A general form of the model is presented and separate expressions for the material, labor and overhead elements of unit cost are derived.

CHAPTER VII

ANALYSIS OF THE SINGLE OPERATION MODEL

Three types of analysis are possible. The first is the sensitivity of the model to random variation in the process variables. The probabilistic model has not yet been posed, and an analysis leading to confidence interval statements from assumptions about the distribution of the random variables can be made at a later time.

The second is the sensitivity of the model to changes in the model parameters. This is appropriate here because a normal cost system treats all factors as parameters, not variables. At the beginning of the year all variables and parameters in the cost system are assigned values based upon sales forecasts, historical data, and estimates of operation parameters for proposed processes. During the year the differences between the normal costs and the actual costs are treated as cost variance. There are material price variances causing material elements of unit cost to vary above or below normal. If a company produces injection molded parts, the plastic resins it purchases would be a major portion of its material costs. Fluctuations in the price of resin are not reflected in normal costs until the end of the year. Another typical source of variation is volume variance. When the sales forecast predicts a demand of 2000 units a week for a

particular part and only half of this demand materializes, the overhead costs are "under-absorbed" and a volume variance occurs. Another is labor variance occurring when wage increases greater than those anticipated at the beginning of the year are granted to labor during the year. Variances such as these are not unusual because it is impossible to anticipate all the possible changes. When changes occur in the favor of the company they also must be explained as variances in the cost report.

For industry in this country wages are a major portion of cost in many processes, and an accurate measurement of labor productivity is necessary. When labor output fails to meet the established standards that are a part of the normal cost, labor "overusage" is the result.

The third type analysis is trade-offs among cost elements. This is the most common type analysis in day-to-day industrial operations. For example, the trend toward automation results from the balancing of labor cost elements against overhead cost elements. Suppose a packaging machine can be purchased commercially that is completely automatic. It will be purchased only if the decrease in labor cost offsets the increase in operating and depreciation expense for the machine. Possibly, packers earn only \$1.50 per hour, and an analysis of the trade-off of labor versus overhead fails to justify the purchase of the machine. A later increase in labor cost to \$1.75 per hour may make the machine

the least cost alternative. Labor, in this sense, prices itself out of business. Since machinery most easily duplicates the unskilled work in a plant, the average skill level that exists in manufacturing is increasing as mechanization gradually eliminates unskilled, low productivity work.

Another common trade-off situation occurs because a policy change or program is started to bring about some type changes in a process variable. An example of this is a value engineering program to reduce unit material costs. The overhead costs of the program are to be balanced by a reduction in purchased material costs. Programs to improve quality levels such as "zero defects" or quality control technique applications are balanced by a decrease in p_i , the fraction defective in the process output. If acceptance sampling plans are installed there is an increase in overhead (indirect), labor and administrative expense. These increases must be offset by a decrease in r_i and t_i which measure the fraction defective in the input materials.

The model that has been developed for the single operation is linear and can be partitioned into the elements of cost. Equations (5), (6) and (7) present expressions for the elements, and we will first take derivatives of these expressions with respect to each of the variables and parameters in order to determine the rate of change in c_i , the unit cost. In making this analysis each parameter is treated as a variable, and the model is analyzed for one variable at a time.

Rather than show all the computations, the derivatives of the material, labor and overhead elements with respect to each model parameter are shown in Tables 3, 4 and 5, respectively. The numerical values used throughout this discussion are shown in Table 2 on page 66. If these values are substituted into Equation 5 for the material portion of the unit cost, the result is \$3.989, as shown in the last row of Table 3.

The derivatives of c_i^M with respect to c_{i-1} , r_i , m_i , t_i and p_i are shown in the table and evaluated for the given data values. In this table the rates of change are given in terms of a unit change in the parameter value. However, a number of parameters in the model are fractional values. If Δp_i were +1 then unit cost would increase by +4.16 as a result of Δp_i . Since such a change is not possible we would be more interested in a Δp_i value of $\Delta p_i = 0.01$ which would result in a change in c_i^M of +0.0416. The same idea applies to the other fractional values in all three tables. In Table 3 a unit change in r_i results in a change of \$1.69 in c_i^M , and for $\Delta r_i = -0.01$ the change would be $\Delta c_i^M = -\$0.0169$.

The same comments apply also to the parameters K_i since a unit change for K_i is very small. In the given data $K_i = 700$ units/shift. A change of 100 units would be more meaningful than the one unit change. Since in Table 4 for the labor elements, $\Delta c_i^L = -\$0.00005$ per unit per unit increase in K_i , a $\Delta K_i = +100$ units results in $\Delta c_i^L = \$0.005/\text{unit}$.

Table 2. Process Parameters Used to Provide Numerical Values in the Sensitivity Analysis of the Model

System Parameter	Symbol	Units	Operation i
Operation Capacity	K_i	units/shift	700
Capacity Factor	b_i	-	1.00
Scrap Loss Factor	p_i	-	0.04
Downtime Factor	d_i	-	0.06
Worked Item	c_{i1}	\$/unit	1.62
Reject Factor	r_{i1}	-	0.03
Reject Factor	r_{i3}	\$/unit	0.04
Purchased Item	m_{i1}	\$/unit	2
Reject Factor	t_{i1}	-	0.08
Total Daywork Labor	G_i	\$/shift	24
Effort Level	Ef_i	-	1.20
Standard Minutes	sm_i	min./unit	0.69
Total Base Rate	br_i	\$/hour	3.00
Total Variable Overhead	k_i	\$/unit	0.40
Total Fixed Overhead	f_i	\$/shift	3000
Total Semifixed Overhead	s_i	\$/shift	1200

Table 3. Rates of Change for Material Element
of Unit Cost Using Equation (5)

Factors: c_{i-1} , r_i , m_i , t_i , p_i

No.	Factor	Rate of Change of c_i^M with respect to Factor	Change in c_i^M per unit change in the Factor
(1)	c_{i-1}	$+\frac{1+r_i}{1-p_i}$	+ 1.07
(2)	r_i	$+\frac{c_{i-1}}{1-p_i}$	+ 1.69
(3)	m_i	$+\frac{1+t_i}{1-p_i}$	+ 1.13
(4)	t_i	$+\frac{m_i}{1-p_i}$	+ 2.08
(5)	p_i	$+\frac{c_{i-1}(1+r_i) + m_i(1+t_i)}{(1-p_i)^2}$	+ 4.16

Equation (5):
$$c_i^M = \frac{c_{i-1}(1+r_i) + m_i(1+t_i)}{1-p_i}$$

Normal Cost:
$$c_i^M = \$3.989/\text{unit}$$

Table 4. Rates of Change for Labor Elements
of Unit Cost Using Equation (6)

Factors: K_i , p_i , d_i , G_i , sm_i , br_i

No.	Factor	Rate of Change of c_i^L with respect to Factor	Change in c_i^L per unit change in the Factor
(6)	K_i	$-\frac{G_i}{K_i^2 b_i (1-p_i) (1-d_i)}$	- 0.00005
(7)	p_i	$+\frac{G_i}{K_i b_i (1-d_i) (1-p_i)^2}$	+ 0.0396
(8)	d_i	$+\frac{G_i}{K_i b_i (1-p_i) (1-d_i)^2}$	+ 0.0404
(9)	G_i	$+\frac{1}{K_i b_i (1-p_i) (1-d_i)}$	+ 0.0016
(10)	sm_i	$+\frac{br_i}{60(1-p_i) (1-d_i)}$	+ 0.0554
(11)	br_i	$+\frac{sm_i}{60(1-p_i) (1-d_i)}$	+ 0.0127

$$\text{Equation (6): } c_i^L = \frac{G_i}{K_i b_i (1-d_i) (1-p_i)} = \frac{(sm_i) (br_i)}{60(1-p_i) (1-d_i)}$$

$$\text{Normal Cost: } c_i^L = \$0.038/\text{unit}$$

Table 5. Rates of Change for Overhead Elements
of Unit Cost Using Equation (7)

Factors: $K_i, p_i, d_i, k_i, f_i, s_i$

No.	Factor	Rate of Change of c_i^O with respect to Factor	Change in c_i^O per unit change in the Factor
(12)	K_i	$-\frac{f_i + s_i}{K_i^2 b_i (1-p_i)(1-d_i)}$	- 0.0095
(13)	p_i	$+\frac{K_i b_i (1-d_i) k_i + f_i + s_i}{K_i b_i (1-d_i)(1-p_i)^2}$	+ 7.36
(14)	d_i	$+\frac{f_i + s_i}{K_i b_i (1-p_i)(1-d_i)^2}$	+ 7.07
(15)	k_i	$+\frac{1}{1-p_i}$	+ 1.04
(16)	f_i	$+\frac{1}{K_i b_i (1-p_i)(1-d_i)}$	+ 0.0016
(17)	s_i	$+\frac{1}{K_i b_i (1-p_i)(1-d_i)}$	+ 0.0016

Equation (7): $c_i^O = \frac{K_i b_i (1-d_i) k_i + f_i + s_i}{K_i b_i (1-d_i)(1-p_i)}$

Normal Cost: $c_i^O = \$7.066/\text{unit}$

For the overhead element of unit cost, a $\Delta K_i = +100$ units results in a decrease in unit cost of $\Delta c_i^O = -\$0.95/\text{unit}$. The elements are additive so that $c_i = c_i^M + c_i^L + c_i^O$ and $\Delta c_i = \Delta c_i^M + \Delta c_i^L + \Delta c_i^O$. This means that given an initial unit cost, c_i , of \$11.092/unit, a change in capacity of $\Delta K_i = +100$ results in a change $\Delta c_i = 0 - \$0.005/\text{unit} - \$0.95/\text{unit}$, or $\Delta c_i = -\$0.955/\text{unit}$. The change in operation capacity for this set of data has a vastly greater affect upon the overhead element of cost; so the sensitivity of the unit cost to capacity changes depends almost entirely upon the overhead element.

To the cost accountant this is interpreted as an increased "absorbtion" of overhead or "factory burden" by additional output.

Summary

Three different kinds of analysis are considered in this chapter. The sensitivity of unit cost to each model parameter is determined for each parameter in the model for each element of cost. The meaning of the results of the sensitivity analysis is then discussed.

CHAPTER VIII

MODELS FOR SEQUENCES OF OPERATIONS

Series Sequence of Independent Operations

Suppose that instead of one operation the process consists of a sequence of two or more operations. The simplest arrangement of this kind is an "in-line" or series sequence illustrated in Figure 12 for two operations. The result is a production system. It is important to consider whether or not the two operations are independent of each other. Independence can be built into the system by placing an in-process storage unit between the two operations. In an independent system such as Figure 12 the unit cost at operation 1 can be written in the basic form as;

$$y_1 = \frac{A_1 X_1 + B_1}{X_{g1}}$$

The output from operation 1 is stored and operation 2 draws worked material from the storage. Then the unit cost at operation 2 is;

$$y_2 = \frac{A_2 X_2 + B_2}{X_{g2}}$$

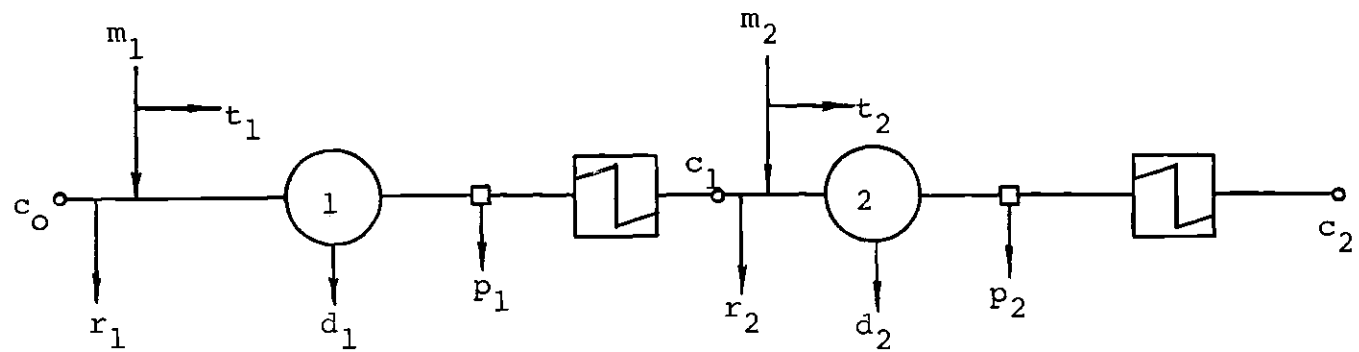


Figure 11. Diagram for a Series Sequence of Two Operations

These two basic forms can be converted to detailed, single operation models giving a separate single operation model for each operation in the sequence of independent operations. The unit cost after operation 1, c_1 , becomes part of the input to c_2 and A_2 depends upon c_1 .

$$A_2 = c_1(1+r_2) + m_2(1+t_2) + k_2$$

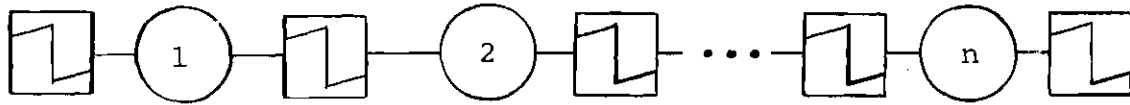
In practice, the computations for unit cost can be made after each operation, and the c_1 value can then be carried into the expression for the next operation in the sequence. An alternative to this procedure is to write one expression to give the unit cost after operation i . It would contain the products of operation parameters such as p_i and d_i for the entire sequence of operations. Since the cost after each operation will be determined anyway, a sequence of calculations using single operation models is the preferred procedure.

The independent sequence of operations can be applied to both discrete and continuous products. Also, this cost model can be written for either daywork or measured work.

Operation Coupling

A series sequence of independent operations is now considered to investigate the interrelationships among the operations. This will show that even though the operations

are independent the unit costs are not. This interrelationship is called operation coupling and shows how an operation in a sequence is related to previous operations and how it will be related to operations that follow it in the sequence. To do this, consider the sequence of n operations drawn below.



For operation 1;

$$c_1 = \frac{A_1 X_1 + B_1}{X_{g1}}$$

$$\text{where } A_1 = c_0(1+r_1) + m_1(1+t_1) + k_1$$

$$X_1 = K_1 b_1 (1-d_1)$$

$$B_1 = G_1 + f_1 + s_1$$

$$X_{g1} = G_1 b_1 (1-d_1)(1-p_1)$$

The term A_1 can be written;

$$A_1 = c_0(1+r_1) + A_1^*$$

to show that each value for A_1 except c_0 is a parameter fixed at operation 1. Then writing $X_{g1} = (1-p_1)X_1$ a revised expression for c_1 is;

$$c_1 = \frac{c_0(1+r_1)X_1}{(1-p_1)X_1} + \frac{A_1^*X_1 + B_1}{(1-p_1)X_1}$$

Now let

$$\alpha_1 = \frac{1+r_1}{1-p_1}$$

$$\beta_1 = \frac{A_1^*X_1 + B_1}{(1-p_1)X_1}$$

and finally

$$c_1 = c_0\alpha_1 + \beta_1$$

In a sequence of operations, the unit cost after each operation is;

$$c_1 = c_0\alpha_1 + \beta_1$$

$$c_2 = c_1\alpha_2 + \beta_2$$

$$c_3 = c_2\alpha_3 + \beta_3$$

$$c_4 = c_3^{\alpha_4} + \beta_4$$

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$$c_n = c_{n-1}^{\alpha_n} + \beta_n$$

Unit cost c_1 is a function of c_0 and the operation 1 parameters. Unit cost c_2 is;

$$c_2 = c_1^{\alpha_2} + \beta_2 = (c_0^{\alpha_1 + \beta_1})^{\alpha_2} + \beta_2 = c_0^{\alpha_1 \alpha_2} + \beta_1^{\alpha_2} + \beta_2$$

For unit cost c_3 we have;

$$\begin{aligned} c_3 &= c_2^{\alpha_3} + \beta_3 = (c_0^{\alpha_1 \alpha_2 + \beta_1 \alpha_2 + \beta_2})^{\alpha_3} + \beta_3 \\ &= c_0^{\alpha_1 \alpha_2 \alpha_3} + \beta_1^{\alpha_2 \alpha_3} + \beta_2^{\alpha_3} + \beta_3 \end{aligned}$$

and c_4 will be;

$$c_4 = c_0^{\alpha_1 \alpha_2 \alpha_3 \alpha_4} + \beta_1^{\alpha_2 \alpha_3 \alpha_4} + \beta_2^{\alpha_3 \alpha_4} + \beta_3^{\alpha_4} + \beta_4$$

so the general expression is;

$$c_n = c_0^{\alpha_1 \dots \alpha_n} + \beta_1^{\alpha_2 \dots \alpha_n} + \beta_2^{\alpha_3 \dots \alpha_n} + \dots + \beta_n$$

and it can be written;

$$c_n = c_0 (\alpha_1 \alpha_2 \alpha_3 \dots \alpha_n) + \sum_{k=1}^n \beta_k (\alpha_{k+1} \alpha_{k+2} \alpha_{k+3} \dots \alpha_n)$$

When there are no worked material inputs to an operation, $c_0 = 0$ and only the second term in the expression exists. Thus, the unit costs after each operation in an independent sequence are actually the operation costs multiplied by the product of the fraction of good units from all previous operations in the sequence.

The term "independence" refers to operating independence, but the unit cost for the output of the process is not functionally independent of the effect of other operations.

Series Sequence of Dependent Operations

A sequence of dependent operations can consist of discrete or continuous products and daywork or measured work operations. In general, a production process is of one type; i.e., either all or none of the labor elements will be measured, and the product is discrete or continuous throughout the sequence. If the product does change - say in a bottling or canning operation - the sequence can be separated into discrete and continuous sub-sequences for evaluation.

To visualize the dependent process, consider Figure 11 without an in-process storage. The unit cost after operation 1 will be;

$$y_1 = \frac{A_1 X_1 + B_1}{X_{g1}}$$

and this cost will be an input to operation 2. At this point a model similar to the independent case model can not be used because the products of losses for downtime and scrap do not represent actual process operation. When downtime occurs the entire process must stop because in-process storages do not exist. If there are only two operations the process will stop if either operation is down. Because time is a continuous variable we do not consider the "simultaneous" failure of two or more operations. The important question of downtime, scrap loss and process capacity will be discussed in the next sections. The model applied to a process consisting of a dependent sequence of operations is presented below.

The denominator of the unit cost expression for the process is;

$$[\text{Min}(K_i b_i)] (1 - \sum_i d_i) (1 - \sum_i p_i)$$

The general form of the numerator is $AX + B$;

$$\text{where } A = [c_0(1+r_1) + \sum_{ij} m_{ij}(1+t_{ij}) + \sum_{ij} k_{ij}]$$

$$X = [\text{Min}(K_i b_i)] (1 - \sum_i d_i)$$

$$B = \sum_{ij} (G_{ij} + f_{ij} + s_{ij})$$

In the numerator there is first an input of worked material to the first operation. Then all purchased material inputs and all variable overhead costs are summed for all operations in the sequence. The sum of the input unit cost values are multiplied by the input in units to the operation to determine the total cost. Labor, fixed overhead, and semifixed overhead are summed for all operations.

Dependent sequences are treated in costing as if they were one operation. There are a number of examples of this type process. The most common is the paced assembly line, particularly those where one conveyor carries a product past a number of work stations where operators perform manual assembly. Also, chemical processes with continuous outputs have this cost structure.

Downtime For Dependent Operations

Consider the unit interval representing a production shift for a two operation process. The fraction of time the process is down because of operation 1 is called δ_1 and the fraction down for operation 2 is called δ_2 . These fractions, δ_1 and δ_2 , are necessarily disjoint intervals as shown below.



The definition for downtime given for the single operation model requires the following definitions for d_1 and d_2 :

$$d_1 = \frac{\delta_1}{1-\delta_2} \qquad d_2 = \frac{\delta_2}{1-\delta_1}$$

which means that d_i is the proportion of the time when operation i *can be running* that it is down. For the unit interval, suppose the downtime minutes charged to operation i are 0.10 of the total shift minutes, and for operation 2 are 0.08 of the total shift minutes. Then;

$$d_1 = \frac{0.10}{0.92} \qquad \text{and} \qquad d_2 = \frac{0.08}{0.90}$$

The expression for d_1 and d_2 can be written as;

$$\delta_1 = d_1 - d_1 \delta_2$$

$$\delta_2 = d_2 - d_2 \delta_1$$

and by substitution, δ_1 and δ_2 can be expressed in terms of d_1 and d_2 as;

$$\delta_1 = \frac{d_1 - d_1 d_2}{1 - d_1 d_2} \qquad \delta_2 = \frac{d_2 - d_1 d_2}{1 - d_1 d_2}$$

By definition, total downtime would be written $d_T = d_1 + d_2$. However, process records show δ_1 and δ_2 as minutes of downtime for each operation. The total downtime

for a process used in the model is $\delta_1 + \delta_2$ because it is a more practical measure. Then the term d_T is the sum of the δ values for the operations in the process. The question of amount of error introduced by this procedure can be considered, since;

$$\delta_1 + \delta_2 = \frac{d_1 + d_2 - 2d_1d_2}{1 - d_1d_2}$$

The error is then;

$$\begin{aligned} \text{Error} &= d_1 + d_2 - \frac{d_1 + d_2 - 2d_1d_2}{1 - d_1d_2} \\ &= \frac{d_1d_2(2-d_1-d_2)}{1 - d_1d_2} \end{aligned}$$

or in terms of δ values;

$$\text{Error} = \frac{\delta_1\delta_2(2-\delta_1-\delta_2)}{(1-\delta_1)(1-\delta_2)}$$

Table 6 shows a comparison of d_T as $\delta_1 + \delta_2$ and d_T as $d_1 + d_2$ with the computed error for seven different pairs of δ values.

Table 6. A Comparison of Total Downtime by Definition
 (d_1+d_2) and by the Model Approximation $(\delta_1+\delta_2)$

δ_1	δ_2	d_1	d_2	d_1+d_2	$\delta_1+\delta_2$	Error
.01	.01	.0101	.0101	.0202	.0200	.0002
.05	.05	.0526	.0526	.1052	.1000	.0052
.10	.10	.1111	.1111	.2222	.2000	.0222
.20	.20	.2500	.2500	.5000	.4000	.1000
.40	.10	.4444	.1667	.6111	.5000	.1111
.80	.05	.8421	.2500	1.0921	.8500	.2421
.90	.01	.9091	.1000	1.0091	.9100	.0991

This comparison shows that the magnitude of the error increases as the δ values increase. Using the downtime definition it is possible for d_T to be greater than 1. Using $\delta_1 + \delta_2$ this is not possible. Since d_i is usually 0.10 or less in most real processes, the total downtime is defined as $d_T = \delta_1 + \delta_2$ for the model as a practical expedient. These results can be extended to any number of operations without difficulty. In the application of the model in the remainder of this paper the specified d_i values represent fractions of total process time. A downtime occurrence of 12 minutes during a 480 minute shift results in $d_i = 0.025$.

As a note on the practical operation of production systems, it could be stated that even though the downtime question is not presented clearly in present cost accounting procedures, the nature of the interdependence of operations is well understood in practice. In-process storages are provided between operations when it is necessary, and long sequence of dependent operations are unusual in manufacturing processes. Most material handling devices that move materials and products act as storages to provide for variable machine rates and downtime occurrences of at least short duration. Even operators working along paced assembly conveyors can work up and down the line a short distance to compensate for variable cycle times at their own as well as at the preceding and following operations.

Scrap Losses For Dependent Operations

For a single operation the scrap loss is applied to process output to determine the output in good units. If the input is X_i and output is X_i , the good units from operation i are $X_{gi} = X_i(1-p_i)$ since a fraction p_i of the output is defective.

If the good units pass to a second operation in a sequence of operations rather than to a storage, the input is $X_{i+1} = X_i(1-p_i)$, the output is $X_{i+1} = X_i(1-p_i)$, and the number of good units from operation $i + 1$ is then;

$$X_{g(i+1)} = X_i(1-p_i)(1-p_{i+1})$$

In a sequence of n dependent operations, the product $(1-p_1)(1-p_2)(1-p_3) \dots (1-p_n)$ will be applied to the flow of units starting through the operations. Since

$$(1-p_i)(1-p_{i+1}) = 1 - p_i - p_{i+1} + p_i p_{i+1}$$

and the scrap fractions are generally values less than 0.05, the model ignores products of scrap fractions. For two operations;

$$1 - p_i - p_{i+1} - p_i p_{i+1} \approx 1 - \sum p_i$$

as a practical expedient and, as in the downtime computation, this permits the use of process data directly without intermediate calculations.

Process Capacity for Dependent Operations

Although independent operations can have input and output rates that vary from operation to operation, a dependent sequence of operations has a capacity limited to the minimum capacity in the sequence of operations. The principle that applies is the idea of a "bottleneck" operation as is found in an assembly line. Since the idea is well known, only the application of the principle to the unit cost model is considered here. To determine process input for the numerator of the unit cost expression and output in good units for the denominator, the notation $[\text{Min}(K_i b_i)]$ is used. It is to be interpreted as the minimum adjusted (by factor b_i) capacity of all operations in the dependent sequence.

Parallel Operations

One example of parallel operations is shown in Figure 12. Clearly, if two separate products are produced on two separate sequences of operations, the operations can be treated as two separate series sequences of operations.

Another example of parallel operations is shown as Figure 12. The parallel operations are part of a dependent sequence. If an IPS were put after operation 1, operations 2 and 3 would become independent and require no special

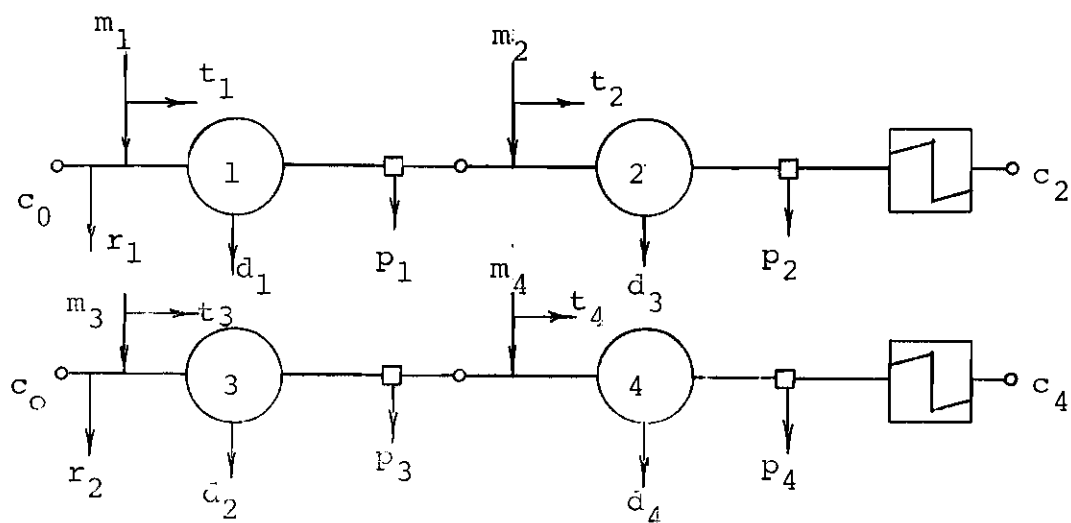


Figure 12. Diagram for Parallel Sequences of Operations

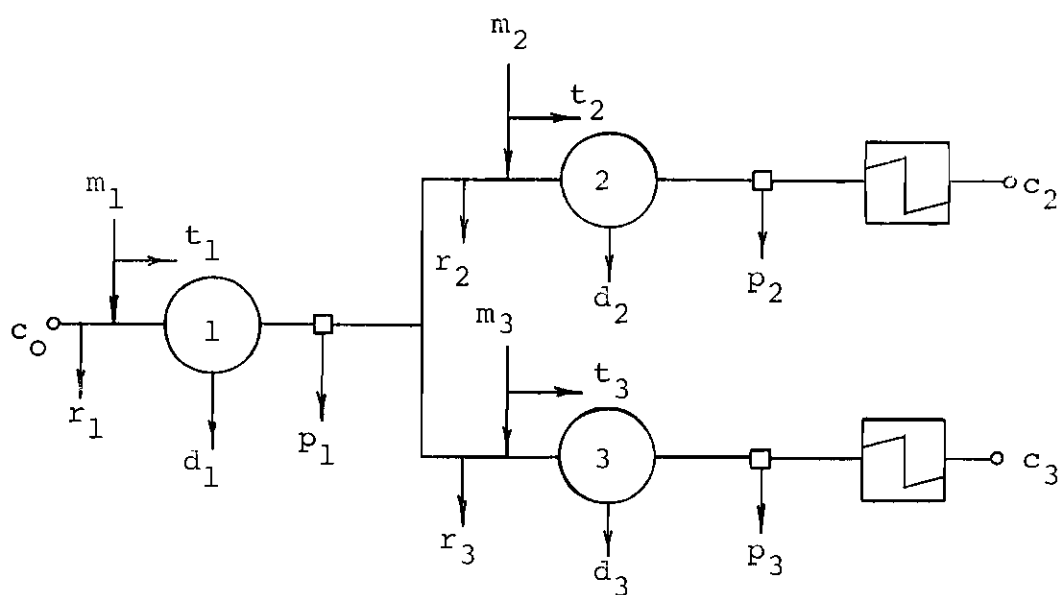


Figure 13. Diagram for a Single to Parallel Process

analysis. However, in Figure 13 there is a dependence of operations 2 and 3 not only on operation 1 but also upon each other. This type of process is called *single to parallel*.

A third example of parallel operations is shown in Figure 14 where there is only one output product and the operations are mutually dependent. This type process is called *parallel to single*.

These two types of parallel operations form the basis for the discussion of all parallel operations. It is assumed that the unit costs for every process containing parallel operations can be determined using a model based upon either one of these two types.

Single to Parallel Model

The process presented as Figure 13 is a single to parallel configuration. There are many possible variations of the basic process idea, but the basic configuration is as shown in Figure 13. To examine this process in detail the process in Figure 13 is presented with some minor modifications as Figure 15.

The modifications are that p_1 , r_2 and r_3 have been eliminated. This means that in the sequence there is no opportunity to inspect the product after operation 1 nor is there an opportunity to inspect the worked material input from operation 1 to operations 2 and 3. Two different products are produced and stored and, therefore, two different

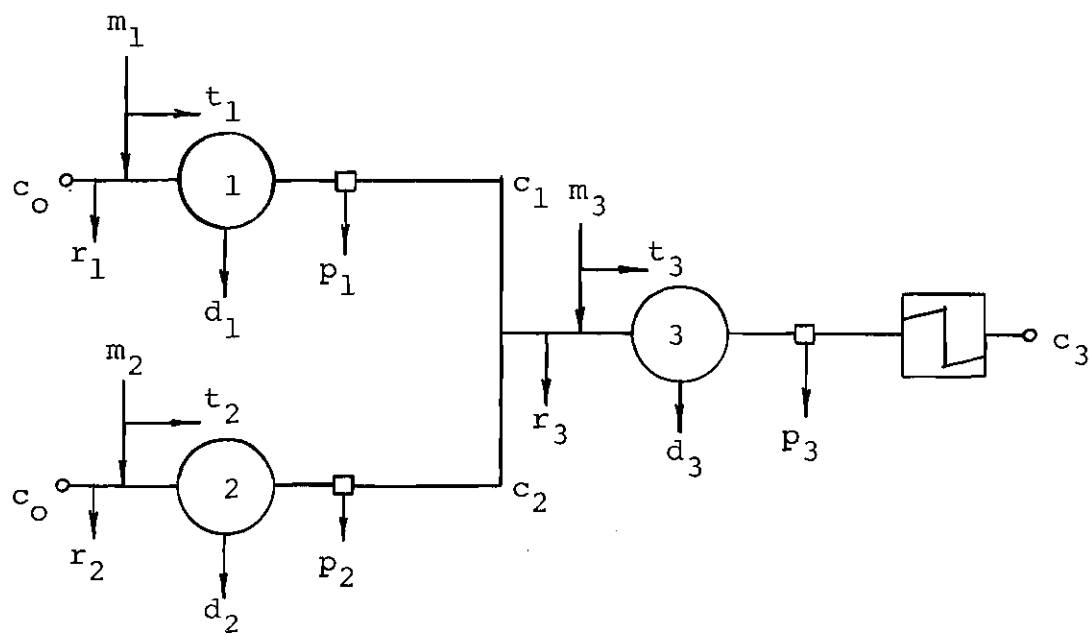


Figure 14. Diagram for a Parallel to Single Process

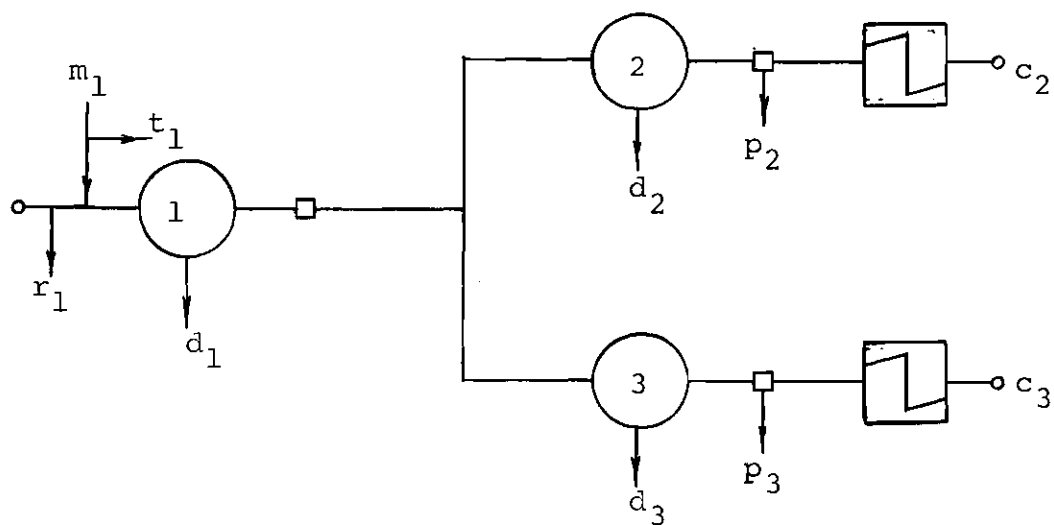


Figure 15. Diagram for a Modified Single to Parallel Process

unit costs are required. In a more general sense, Figure 13 is used with p_1 , r_2 and r_3 equal to zero.

The effect of downtime can be considered in a number of different ways: if operation 1 stops, the entire process stops because operations 2 and 3 have no inputs; another way is to assume that a stop at either operation 2 or 3 does not cause the remaining operations to stop - they continue to operate after an adjustment to the capacities to maintain a balance of inputs and outputs. Other procedures for the operation of the process after downtime occurs can be devised.

Rather than attempt to develop a process model for a range of possible operating procedures, only two cases are considered:

Case 1. All three operations are running and the process is running.

Case 2. Any one of the three operations stops and the entire process stops.

Case 1 is a situation that is considered only to develop the model without the complication of downtime or scrap loss. The model is developed using the definition previously used for a unit cost; i.e., total cost input divided by output in good units. The form for each operation is;

$$Y_i = \frac{A_i X_i + B_i}{X_{gi}}$$

and the numerator is total input cost per day (or shift) determined by multiplying input unit costs by the units of input and adding total cost charges.

For operation 1 we have;

$$y_1 = \frac{A_1 X_1 + B_1}{X_{g1}}$$

$$\text{where } A_1 = c_0(1+r_1) + m_1(1+t_1) + k_1$$

$$X_1 = K_1 b_1 (1-d_1)$$

$$B_1 = G_1 + f_1 + s_1$$

$$X_{g1} = K_1 b_1 (1-d_1) (1-p_1)$$

To illustrate this calculation, assume that $c_0 = 0$ and use the data presented on page 92 for the operation data. Notice in this process that the output from operation 1 is distributed to operations 2 and 3 and, therefore, K_1 must equal $K_2 + K_3$ if the process is balanced. Assuming $d_1 = 0$ and $p_1 = 0$, the total input cost to operation 1 is then;

$$\begin{aligned} A_1 X_1 + B_1 &= [0 + 1.00(1.04) + .50](700) + 24.00 + 420.00 \\ &= 1078 + 24 + 420 = \$1522 \end{aligned}$$

It is possible to write a unit cost after operation 1 as;

$$c_1 = \frac{1522}{700} \text{ dollars per unit}$$

There is no need for c_1 because identifiable products come only from operations 2 and 3. The input to operation 2 is;

$$A_2X_2 + B_2 = [m_2(1+t_2)+k_2](K_2b_2) + G_2 + f_2 + s_2$$

This is input to the entire process (not just operation 2) and, therefore, no term $c_1(1+r_1)$ occurs in this expression. The value for the input is;

$$\begin{aligned} A_2X_2 + B_2 &= [3.50(1+.02) + .20](200) + 32 + 490 \\ &= 754 + 32 + 490 = \$1276 \end{aligned}$$

The input to the process for operation 3 is;

$$\begin{aligned} A_3X_3 + B_3 &= [m_3(1+t_3)+k_3](K_3b_3) + G_3 + f_3 + s_3 \\ &= [0.80(1+.03)+.10](500) + 16 + 1060 \\ &= 462 + 16 + 1060 = \$1538 \end{aligned}$$

The output from operation 1 is 700 units and no scrap loss is measured until after operations 2 and 3. Therefore, p_2 and p_3 , as they are measured, include defects from operation 1. The output in good units from operations 2 and 3 are $200 - 4 = 196$ and $500 - 20 = 480$, respectively.

Table 7. Process Parameters for the Single
to Parallel Model

System Parameter	Symbol	Units	Operation Number			
			1	2	3	4
Operation Capacity	K_i	units/shift	700	200	500	800
Capacity Factor	b_i	-	1.00	1.00	1.00	1.00
Scrap Loss Factor	p_i	-	0.04	0.02	0.04	0.01
Downtime Factor	d_i	-	0.08	0.04	0.04	0.10
Worked Item	c_{i1}	\$/unit	0	0	0	0
Reject Factor	r_{i1}	-	0.02	0.02	0.02	0.03
Purchased Item	m_{i1}	\$/unit	1	3.50	0.80	1.50
Reject Factor	t_{i1}	-	0.04	0.02	0.03	.04
Total Daywork Labor	G_i	\$/shift	24	32	16	32
Total Variable Overhead	k_i	\$/unit	0.50	0.20	0.10	.20
Total Fixed Overhead	f_i	\$/shift	300	400	1000	100
Total Semifixed Overhead	s_i	\$/shift	120	90	60	20

A basic question in single to parallel processes is how the input cost to operation 1 is to be distributed to operations 2 and 3. The method selected here is to use the ratio of total output divided into the number of units distributed to each of the two following operations. The proportion of operation 1 total cost distributed to operation 2 is;

$$\frac{K_2 b_2}{K_1 b_1} [A_1 X_1 + B_1] \quad \text{or} \quad \frac{K_2 b_2}{K_1 b_1} [A_1 (K_1 b_1) + B_1]$$

and to operation 3 is;

$$\frac{K_3 b_3}{K_1 b_1} [A_1 X_1 + B_1] \quad \text{or} \quad \frac{K_3 b_3}{K_1 b_1} [A_1 (K_1 b_1) + B_1]$$

so assuming $d_1 = d_2 = d_3 = 0$, these values are;

$$c_2 = \frac{\frac{K_2 b_2}{K_1 b_1} [A_1 K_1 b_1 + B_1] + A_2 X_2 + B_2}{X_{g2}}$$

$$c_2 = \frac{\frac{K_2 b_2}{K_1 b_1} [A_1 K_1 b_1 + B_1] + A_2 X_2 + B_2}{K_2 b_2 (1 - p_2)}$$

and

$$c_3 = \frac{\frac{K_3 b_3}{K_1 b_1} [A_1 K_1 b_1 + B_1] + A_3 K_3 b_3 + B_3}{K_3 b_3 (1 - p_3)}$$

Using the numerical data, these unit costs are;

$$c_2 = \frac{\frac{200}{700} [1522] + 1276}{200 - 4} = \$8.73 \text{ per unit}$$

and

$$c_3 = \frac{\frac{500}{700} [1522] + 1538}{500 - 20} = \$5.47 \text{ per unit}$$

For Case 2, the entire process stops whenever any one of the three operations stops. The model in this case represents actual process operation as long as the d_i values recorded for downtime assign to only one operation the responsibility for each stop. The input cost to operation 1 is;

$$\begin{aligned} c_1 X_{g1} &= A_1 X_1 + B_1 = A_1 K_1 b_1 (1 - \sum_{i=1}^3 d_i) + B_1 \\ &= [c_0 (1 + r_1) + m_1 (1 + t_1) + k_1] K_1 b_1 (1 - \sum_{i=1}^3 d_i) + B_1 \end{aligned}$$

Assuming that $c_0 = 0$ as before, and with $\sum_{i=1}^3 d_i = 0.08 + 0.04 + 0.04 = 0.16$, this total input cost is;

$$\begin{aligned} & [0+1(1+.04)+.50]700(.84) + 24 + 420 \\ & = 588[1+.04+.50] + 24 + 420 = \$1349.52 \end{aligned}$$

For operation 2 the total input cost is;

$$\begin{aligned} & K_2 b_2 (1 - \sum_{i=1}^3 d_i) [m_2 (1+t_2) + k_2] + G_2 + f_2 + s_2 \\ & = 200(.84) [3.50(1+.02) + .20] + 32 + 490 = \$1155.36 \end{aligned}$$

and for operation 3;

$$\begin{aligned} & K_3 b_3 (1 - \sum_{i=1}^3 d_i) [m_3 (1+t_3) + k_3] + G_3 + f_3 + s_3 \\ & = 500(.84) [.80(1+.03) + .10] + 16 + 1060 = \$1464.08 \end{aligned}$$

Then once again the cost model takes the form;

$$c_2 = \frac{\frac{200}{700}[1349.52] + 1155.36}{200(1 - \sum d_i)(1 - p_2)} = \frac{1540.94}{164.64} = \$9.36/\text{unit}$$

and

$$c_3 = \frac{\frac{500}{700}[1349.52] + 1464.08}{500(1-d_1)(1-p_3)} = \frac{2428.02}{403.20} = \$6.02/\text{unit}$$

Parallel to Single Model

The process presented on page 88 as Figure 14 is the parallel to single configuration. The development of this model follows almost identically the development of the single to parallel model. In this dependent sequence there is only one output product after operation 3. There are no IPS's after operations 1 and 2 and no inspection after operations 1 and 2. The worked material inputs to operations 1 and 2 are inspected before operations 1 and 3, but the inputs from operations 1 and 2 to operation 3 are not inspected. Only one p value is recorded, p_3 , and it is the result of defects produced at all three operations. With this arrangement it is necessary to balance the outputs from operations 1 and 2 to the input to operation 3. The parameter values given in Table 8 on page 97 are used in the examples for this model with $K_1 = 200$, $K_2 = 300$ and $K_3 = 500$.

As in the previous model, only two types of process operation are considered:

Case 1. All three operations are running and the process is running.

Case 2. Any one of the three operations stops and the entire process stops.

Table 8. Process Parameters for the Parallel
to Single Model

System Parameter	Symbol	Units	Operation Number			
			1	2	3	4
Operation Capacity	K_i	units/shift	200	300	500	800
Capacity Factor	b_i	-	1.00	1.00	1.00	1.00
Scrap Loss Factor	p_i	-	0.04	0.02	0.04	0.01
Downtime Factor	d_i	-	0.08	0.04	0.04	0.10
Worked Item	c_{il}	\$/unit	0	0	0	0
Reject Factor	r_{il}	-	0.02	0.02	0.02	0.03
Purchased Item	m_{il}	\$/unit	1	3.50	0.80	1.50
Reject Factor	t_{il}	-	0.04	0.02	0.03	0.04
Total Daywork Labor	G_i	\$/shift	24	32	16	32
Total Variable Overhead	k_i	\$/unit	0.50	0.20	0.10	0.20
Total Fixed Overhead	f_i	\$/shift	300	400	900	100
Total Semifixed Overhead	s_i	\$/shift	120	90	160	20

Since Case 1 is a no downtime model and never occurs in practice, it is not considered any further. As in the previous section, it is Case 2 for which the model is developed.

Total cost inputs to operation 1 are;

$$A_1 X_1 = [c_0(1+r_1) + m_1(1+t_1) + k_1] X_1$$

where

$$X_1 = K_1 b_1 (1 - \sum_{i=1}^3 d_i)$$

To operation 2 the inputs are;

$$A_2 X_2 = [c_0(1+r_2) + m_2(1+t_2) + k_2] X_2$$

where

$$X_2 = K_2 b_2 (1 - \sum_{i=1}^3 d_i)$$

Since there is no inspection after operations 1 and 2, the outputs in units to operation 3 are the same X_1 and X_2 values. Since both outputs go to operation 3 a single unit cost can be determined as the input unit cost to operation 3. From operation 1 the total cost input to operation 3 is

$A_1X_1 + B_1$, and for operation 2 it is $A_2X_2 + B_2$. The total cost of the output of the two operations is;

$$A_1X_1 + A_2X_2 + B_1 + B_2$$

The unit cost after operation 3, c_3 , is computed by adding all input costs from operations 1, 2 and 3 and dividing by the output in good units from operation 3. In general terms this is;

$$c_3 = \frac{A_1X_1 + A_2X_2 + A_3X_3 + B_1 + B_2 + B_3}{X_{g3}}$$

The details are;

$$A_1X_1 + B_1 = [c_0(1+r_1)+m_1(1+t_1)+k_1](X_1) + G_1 + f_1 + s_1$$

$$A_2X_2 + B_2 = [c_0(1+r_2)+m_2(1+t_2)+k_2](X_2) + G_2 + f_2 + s_2$$

$$A_3X_3 + B_3 = [c_0(1+r_3)+m_3(1+t_3)+k_3](X_3) + G_3 + f_3 + s_3$$

Examples for the calculation of these values are shown as follows:

$$\begin{aligned}
 A_1X_1 + B_1 &= [0(1+.02)+1(1+.04)+.50]200(1-.16) + 24 + 420 \\
 &= [1.54]168 + 444.00 \\
 &= 258.72 + 444.00 = \$702.72
 \end{aligned}$$

$$\begin{aligned}
 A_2X_2 + B_2 &= [0(1+.02)+3.50(1+.02)+.20]300(1-.16) + 32 + 490 \\
 &= [3.77]252 + 32 + 490.00 \\
 &= 950.04 + 522.00 = \$1472.04
 \end{aligned}$$

$$\begin{aligned}
 A_3X_3 + B_3 &= [.80(1+.03)+.10]500(1-.16) + 16 + 1060 \\
 &= [.924]420 + 1076.00 \\
 &= 388.08 + 1076.00 = \$1464.08
 \end{aligned}$$

Since $X_{g3} = 500(1 - \sum_{i=1}^3 d_i)(1-p_3) = 500(.84)(.96) = 403.20$ in good output units from operation 3, the unit cost, c_3 , is;

$$c_3 = \frac{702.72 + 1472.04 + 1464.08}{403.20} = \frac{3638.84}{403.20} = \$9.025/\text{unit}$$

In single to parallel operations the total cost per shift incurred by the good units of output is distributed to the two operations that follow the single operation according to the proportion of the output distributed to each of the parallel operations. The same would be true for any number of following operations (from 1 to n).

Parallel to single operations, on the other hand, require that the total costs of all operations supplying the single operation be added to the cost per shift of the single

operation, and the overall total is absorbed by the good units of output from that one operation.

In both cases, the procedure requires that total costs be determined according to a clearly defined method. The detailed cost model used is written below, although the costs would usually be determined in practice as was done in the example; i.e., the total cost by operation would be determined and then summed. If a computer is used the cost would be programmed for the entire process as is given below;

$$c_3 = \frac{A_1X_1 + A_2X_2 + A_3X_3 + B_1 + B_2 + B_3}{X_{g3}} \quad (8)$$

where the numerator terms are as shown on page 100, and

$$X_{g3} = K_3 b_3 (1 - \sum_{i=1}^3 d_i) (1 - \sum_{i=1}^3 p_i)$$

Summary

Sequences of operations are considered here, and unit cost models are developed for series and parallel system components. Dependence and independence are considered for unit cost models. The downtime, scrap loss and capacity factors for operations in sequences are presented, and principles for applying these factors to complex models are formulated.

CHAPTER IX

MODELS FOR PRODUCTION SYSTEMS

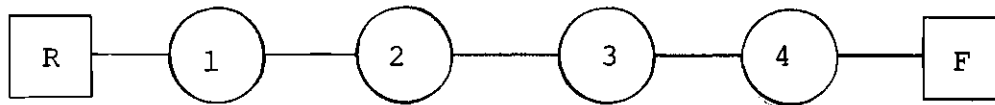
The Reduced Form for Production Systems

At this point, models have been developed to determine unit costs for the following configurations in the manufacturing process:

- (1) the single operation;
- (2) the series sequence of operations;
- (3) single to parallel sequences;
- (4) parallel to single sequences.

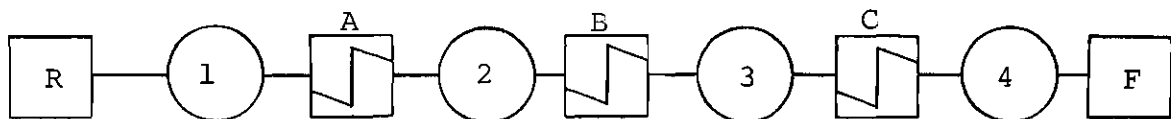
It is now possible to apply these cost models to manufacturing systems to show how they can be used in process analysis and process design. A problem that will be encountered is the wide variety of possible configurations of any set of operations. To simplify the method of analysis, a canonical form can be devised for production processes. The procedure for analysis is: (1) the real world process is analyzed to determine the process characteristics and operation parameters; (2) the process is diagramed and the parameters are tabulated; (3) the process is reduced to a canonical form; and (4) the unit cost for the process is determined.

To develop the reduced form, consider a four operation process as shown below.



Because the process always begins with raw materials (or worked materials), an initial inventory is shown with an "R" to indicate a raw material inventory. The output of the process is to a finished goods inventory marked "F." Within the process, unit costs are determined only at inventory points. The costs of raw materials are known, and it is possible to make physical counts of the inventory, R, to determine the quantities on hand. The same is true for the finished goods inventory, F. In fact, at any inventory point a cost with a countable, meaningful, physical unit is determinable. This is not always the case between operations. The principle followed is that unit costs can be determined only at inventory points.

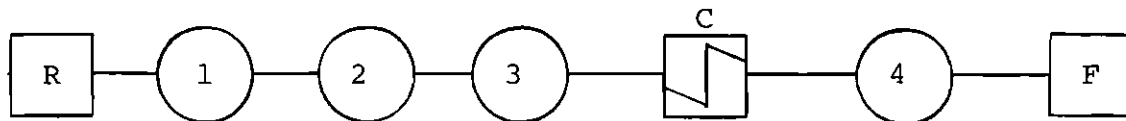
A second diagram of the same four operation process is shown below.



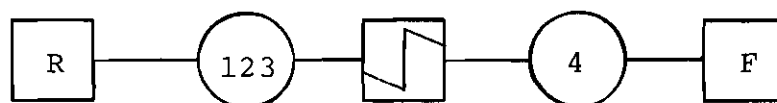
In this diagram, in-process storages (IPS's) are placed between each pair of operations and labeled A, B and C. Since this is a single product process there is no IPS between operation 4 and storage F, and no IPS between storage R and operation 1.

For any process with n operations there can be, at most, $(n-1)$ IPS's. The process shown at the bottom of page 103 is in its most reduced form. The costs for the process can be determined at A, B, C and F. The costs would be determined by four applications of the single operation model.

Suppose IPS's A and B are removed from the process, as shown below.

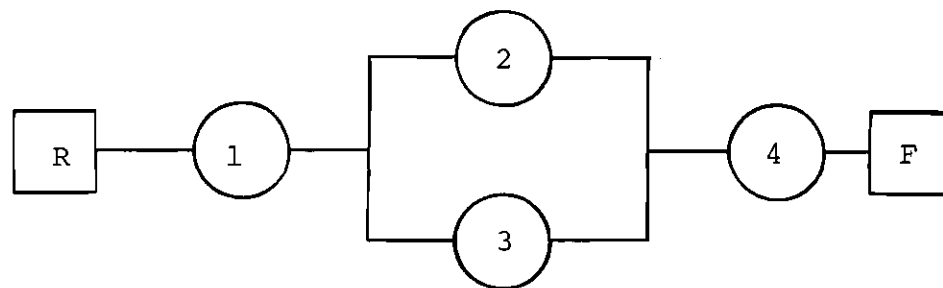
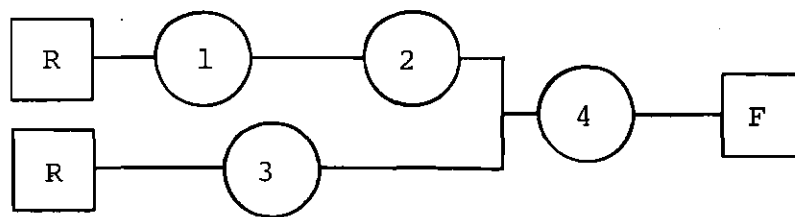


The costs would now be determined at storages C and F by treating operations 1, 2 and 3 as a series sequence and operation 4 as a single operation. In order to determine the cost the process diagram is reduced to the form shown below.



For a series sequence the reduction consists of changing each dependent sequence of operations into independent, single operation sequences. In a four-operation process there are three possible locations for IPS's. At each possible location for an IPS there either is one IPS or none; therefore, 2^3 possible line configurations exist.

A four operation process producing one product could have many different configurations. A series sequence is one possibility, and two others are shown below.



In both diagrams the number of operations is four and the number of possible IPS locations is three. Either configuration can be evaluated 2^3 ways, depending upon whether or not IPS's are actually put at those locations. In both

cases, since there are no IPS's, the processes can be reduced to the canonical form shown below



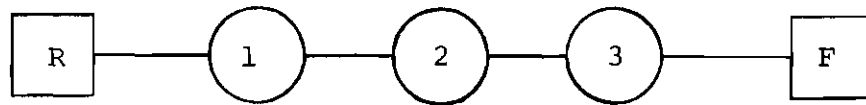
One unit cost, at storage F, would be computed for the process as c_4 .

Even though process configurations can be reduced for calculation to a canonical form, the equations representing a given process have to be written individually because the existence of inspection points either before or after an operation depends upon management decisions rather than any prescribed process form.

The Location of In-Process Storage Units

If a manufacturing process is composed of a fixed number of operations arranged in a specified sequence, variations in the application of unit cost models to the process are possible. A unit cost is measured at each storage point in the process. For each different arrangement of IPS units there is a different set of model equations. A question is whether or not a change in the location of IPS units can change output product unit costs.

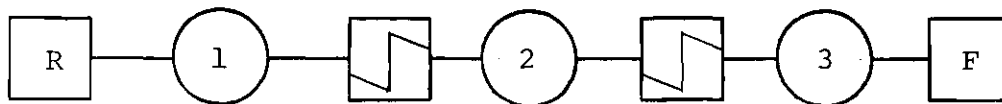
Suppose we have the series sequence of three operations given on page 107.



The symbol R represents raw material storage and F represents finished goods storage.

There are only two possible locations for IPS in this process: between operations 1 and 2 or between operations 2 and 3. The IPS locations can be described by a "0" if an IPS is not placed at a possible location, and a "1" if it is. Then, in the diagram above, the arrangement is 00. The other possible IPS arrangements are 01, 10 and 11. In total there are 2^{n-1} locations among a set of n operations, and in this case 2^2 or the above four (00, 01, 10, 11).

There are two reasons why a change in IPS location will affect unit cost: one downtime and the other the process output. To show this, consider the 00 process drawn above and the 11 process given below.



By writing the cost equations for each process, the difference is clear.

For 00:

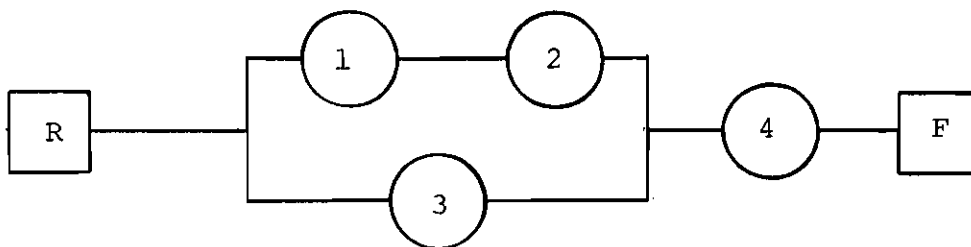
$$c_3 = \frac{1}{[\text{Min}(K_i b_i)] (1 - \sum_{i=1}^3 d_i) (1 - \sum_{i=1}^3 p_i)} \quad [. . .]$$

and for 11:

$$c_3 = \frac{1}{K_3 b_3 (1 - d_3) (1 - p_3)} \quad [. . .]$$

On the one hand, the process output is the minimum of the three operation capacities, and on the other the operation capacities are independent of each other because of IPS effects. The same analysis applies to downtime. In the 00 case, a stoppage at any one operation stops all three operations, and in the 11 case only the operation down is affected. The p_i value is treated as a sum, as is d_i in the 00 case, but the p_i values result in the same total loss to process output in either case.

To illustrate the change in unit cost with change in IPS location, the four operation parallel-series process shown below will be completely analyzed.



The parameter values assumed for this process are shown on page 110. All possible IPS locations are summarized by operation output and operation unit cost in Table 10 on page 111. An analysis of Table 10 shows that minimum unit cost occurs for a full set of IPS units, the expected result. The same process with no IPS units has a unit cost of \$11.91 above the minimum value, and this difference is due only to use of IPS. Therefore, the difference can be used to determine the break-even cost for IPS units.

If only one IPS unit were proposed for this process, the locations 100, 010 and 001 would be used to determine a best storage location. In this case it would be 010 or after operation 2.

In order to apply this type analysis, the details of the application of unit cost models to the process are shown completely in the remainder of this section.

IPS Location - 000

Process.

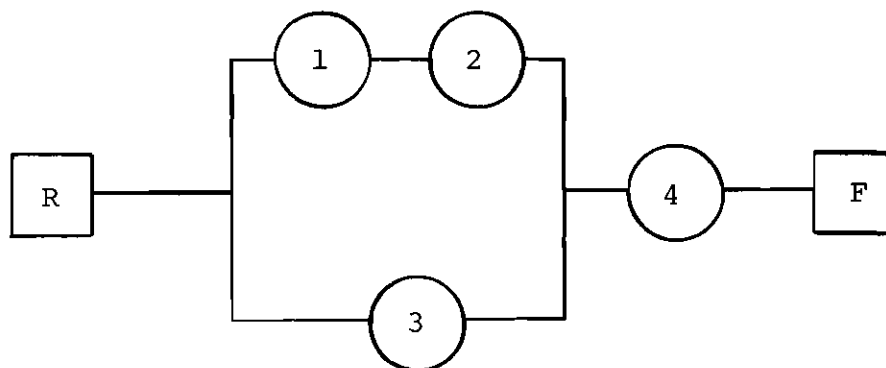


Table 9. Parameter Values for the Analysis of IPS Location

System Parameter	Symbol	Units	Operation Number			
			1	2	3	4
Operation Capacity	K_i	units/day	600	1000	800	800
Capacity Factor	b_i	-	.9	.8	.9	.8
Scrap Loss Factor	p_i	-	.06	.03	.04	.02
Downtime Factor	d_i	-	.03	.01	.02	.01
Worked Item	c_i	\$/unit	**	**	**	**
Reject Factor	r_i	-	.01*	.01*	.03*	.02*
Purchased Item	m_i	\$/unit	4.00	20.00	60.00	12.00
Reject Factor	t_i	-	.01	.04	.04	.03
Total Daywork Labor	G_i	\$/day	60	100	40	40
Total Variable Overhead	k_i	\$/unit	.40	.10	.70	.20
Total Fixed Overhead	f_i	\$/day	300	1000	400	200
Total Semifixed Overhead	s_i	\$/day	100	50	150	200

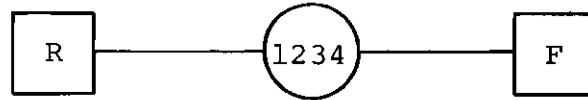
* These factors apply only if worked material comes to the operation from an IPS. Otherwise the factor is "0."

** These costs are computed unit costs from previous operations.

Table 10. A Summary of Outputs and Unit Costs for Every Possible IPS Location in a Four Operation Process

IPS Location	Operation 1		Operation 2		Operation 3		Operation 4	
	Output (units)	Unit Cost (\$)	Output	Unit Cost	Output	Unit Cost	Output	Unit Cost
000	*	*	*	*	*	*	427	124.92
100	492	5.66	*	*	*	*	559	116.24
010	*	*	472	31.22	*	*	584	116.09
001	*	*	*	*	678	66.58	457	123.26
110	492	5.66	768	28.95	*	*	584	113.62
101	492	5.66	*	*	678	66.58	596	115.33
011	*	*	472	31.22	678	66.58	621	115.37
111	492	5.66	768	28.95	678	66.58	621	113.01

* Costs are calculated only at storage points.

Reduced Process.Equations.

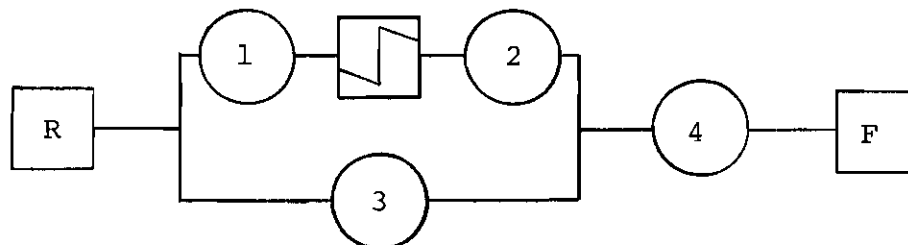
$$c_4 = \frac{X_4 \sum_{i=1}^4 A_i + \sum_{i=1}^4 B_i}{X_{g4}}$$

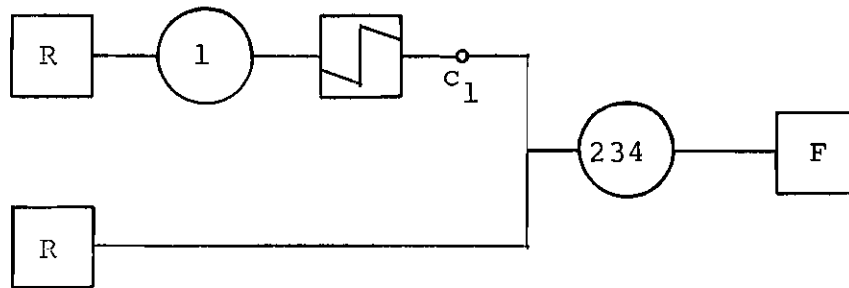
$$\text{where } \sum A_i = c_0(1+r_1) + c_0(1+r_3) + \sum_{i=1}^4 m_i(1+t_i) + \sum_{i=1}^4 k_i$$

$$X_4 = [\text{Min}(K_i b_i)] (1 - \sum_{i=1}^4 d_i)$$

$$\sum B_i = \sum_{i=1}^4 (G_i + f_i + s_i)$$

$$X_{g4} = [\text{Min}(K_i b_i)] (1 - \sum_{i=1}^4 d_i) (1 - \sum_{i=1}^4 p_i)$$

IPS Location - 100Process.

Reduced Process.Equations.

$$c_1 = \frac{A_1 X_1 + B_1}{X_{g1}}$$

$$\text{where } A_1 = c_0(1+r_1) + m_1(1+t_1) + k_1$$

$$X_1 = K_1 b_1 (1-d_1)$$

$$B_1 = G_1 + f_1 + s_1$$

$$X_{g1} = K_1 b_1 (1-d_1) (1-p_1)$$

$$c_4 = \frac{X_4 \sum_{i=2}^4 A_i + \sum_{i=2}^4 B_i}{X_{g4}}$$

$$\text{where } \sum_{i=2}^4 A_i = c_0(1+r_3) + c_1(1+r_2) + \sum_{i=2}^4 m_i(1+t_i) + \sum_{i=2}^4 k_i$$

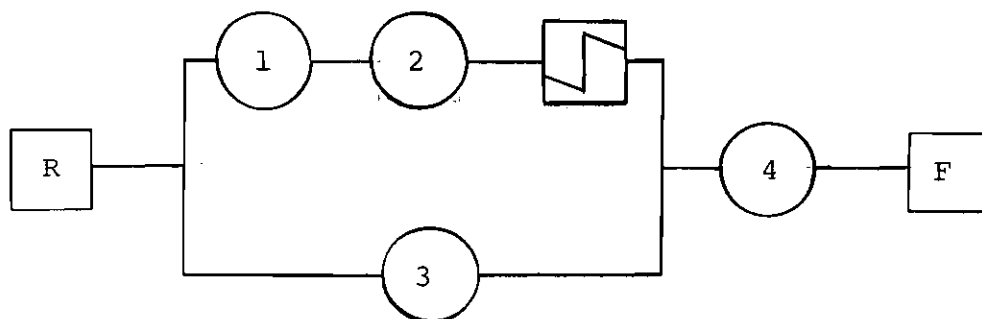
$$X_4 = [\text{Min}(K_i b_i)_{2-4}] (1 - \sum_{i=2}^4 d_i)$$

$$\sum_{i=2}^4 B_i = \sum_{i=2}^4 (G_i + f_i + s_i)$$

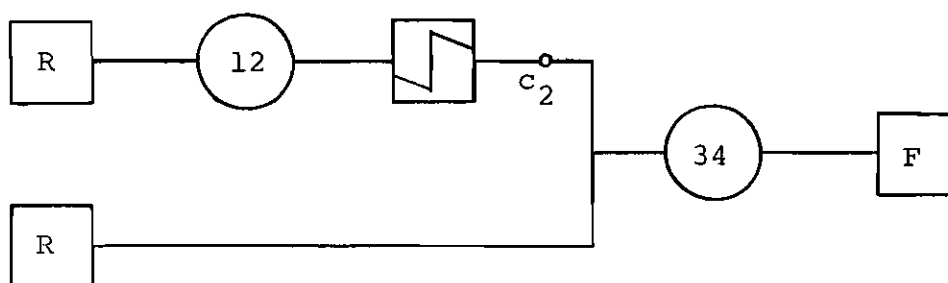
$$X_{g4} = [\text{Min}(K_i, b_i)] (1 - \sum_{i=2}^4 d_i) (1 - \sum_{i=2}^4 p_i)$$

IPS Location - 010

Process.



Reduced Process.



Equations.

$$c_2 = \frac{x_2 \sum_{i=1}^2 A_i + \sum_{i=1}^2 B_i}{x_{g2}}$$

$$\text{where } \sum_{i=1}^2 A_i = c_0(1+r_1) + \sum_{i=1}^2 m_i(1+t_i) + \sum_{i=1}^2 k_i$$

$$X_2 = [\text{Min}(K_i b_i)]_{1,2} (1 - \sum_{i=1}^2 d_i)$$

$$\sum_{i=1}^2 B_i = \sum_{i=1}^2 (G_i + f_i + s_i)$$

$$X_{g2} = [\text{Min}(K_i b_i)]_{1,2} (1 - \sum_{i=1}^2 d_i) (1 - \sum_{i=1}^2 p_i)$$

$$c_4 = \frac{X_4 \sum_{i=3}^4 A_i + \sum_{i=3}^4 B_i}{X_{g4}}$$

$$\text{where } \sum_{i=3}^4 A_i = c_2(1+r_4) + \sum_{i=3}^4 m_i(1+t_i) + \sum_{i=3}^4 k_i$$

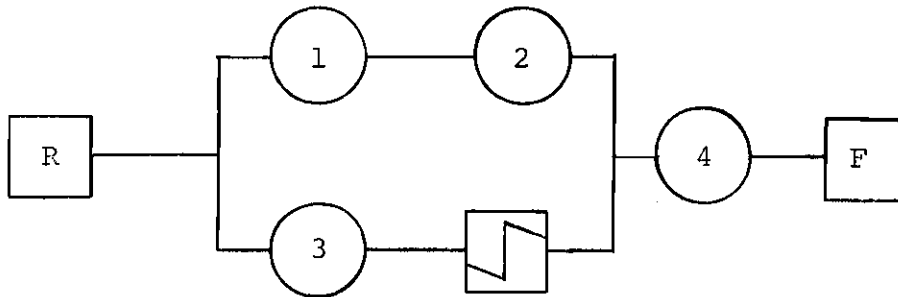
$$X_4 = [\text{Min}(K_i b_i)]_{3,4} (1 - \sum_{i=3}^4 d_i)$$

$$\sum_{i=3}^4 B_i = \sum_{i=3}^4 (G_i + f_i + s_i)$$

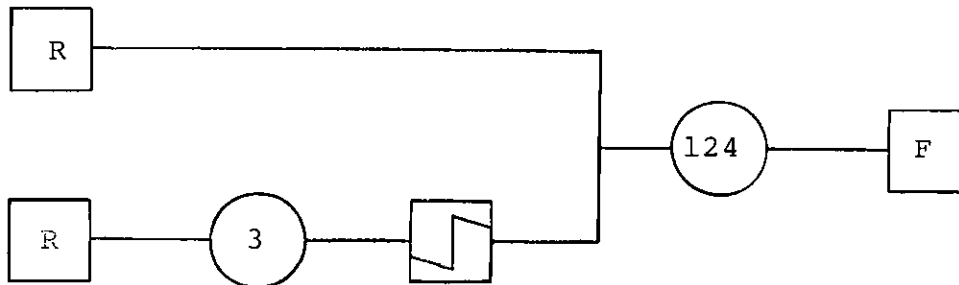
$$X_{g4} = [\text{Min}(K_i b_i)]_{3,4} (1 - \sum_{i=3}^4 d_i) (1 - \sum_{i=3}^4 p_i)$$

IPS Location - 001

Process.



Reduced Process.



Equations.

$$c_3 = \frac{A_3 X_3 + B_3}{X_{g3}}$$

$$\text{where } A_3 = c_0(1+r_3) + m_3(1+t_3) + k_3$$

$$X_3 = K_3 b_3 (1-d_3)$$

$$B_3 = G_3 + f_3 + s_3$$

$$X_{g3} = K_3 b_3 (1-d_3) (1-p_3)$$

$$c_4 = \frac{X_4 \sum_{124} A_i + \sum_{124} B_i}{X_{g4}}$$

$$\text{where } \sum_{124} A_i = c_0(1+r_1) + \sum_{124} m_i(1+t_i) + \sum_{124} k_i$$

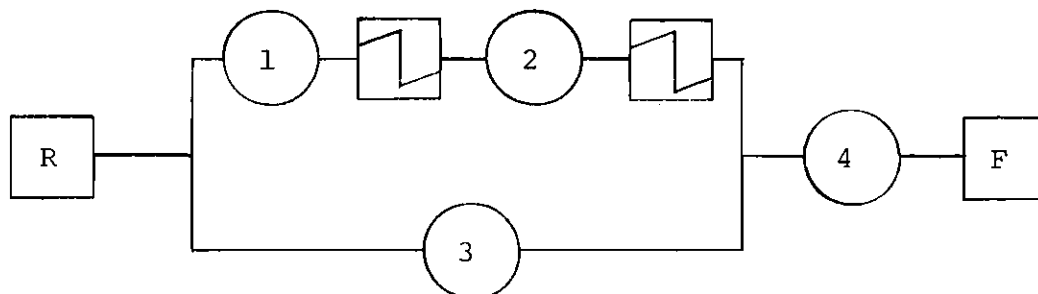
$$X_4 = [\text{Min}(K_i b_i)] (1 - \sum_{124} d_i)$$

$$\sum_{124} B_i = \sum_{124} (G_i + f_i + s_i)$$

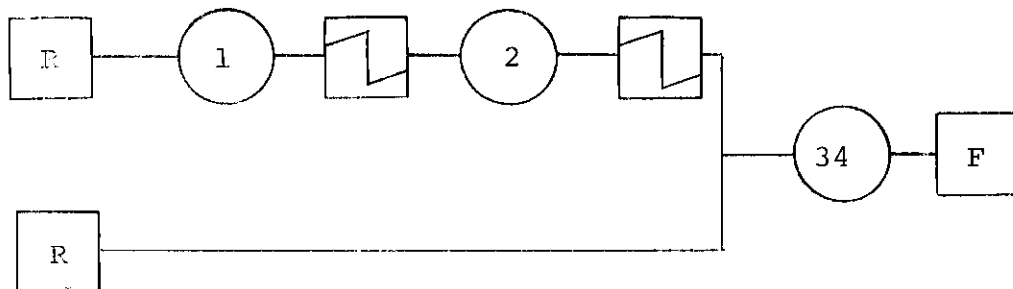
$$X_{g4} = [\text{Min}(K_i b_i)] (1 - \sum_{124} d_i) (1 - \sum_{124} p_i)$$

IPS Location - 110

Process.



Reduced Process.



Equations.

c_1 same as 100 location

$$c_2 = \frac{A_2 X_2 + B_2}{X_{g2}}$$

$$\text{where } A_2 = c_1(1+r_2) + m_2(1+t_2) + k_2$$

$$X_2 = K_2 b_2 (1-d_2)$$

$$B_2 = G_2 + f_2 + s_2$$

$$X_{g2} = K_2 b_2 (1-d_2) (1-p_2)$$

$$c_4 = \frac{X_4 \sum_{i=3}^4 A_i + \sum_{i=3}^4 B_i}{X_{g4}}$$

$$\text{where } \sum_{i=3}^4 A_i = c_0(1+r_3) + c_2(1+r_4) + \sum_{i=3}^4 m_i(1+t_i) + \sum_{i=3}^4 k_i$$

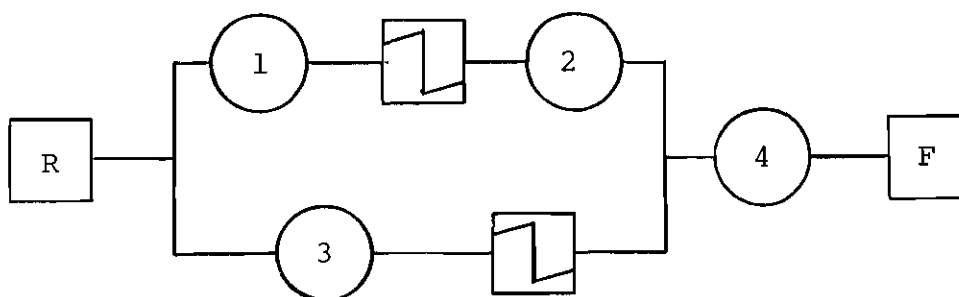
$$X_4 = [\text{Min}(K_i b_i)_{3,4}] (1 - \sum_{i=3}^4 d_i)$$

$$\sum_{i=3}^4 B_i = \sum_{i=3}^4 (G_i + f_i + s_i)$$

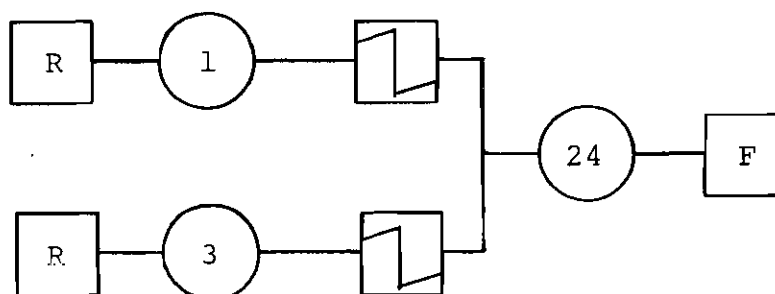
$$X_{g4} = [\text{Min}(K_i b_i)_{3,4}] (1 - \sum_{i=3}^4 d_i) (1 - \sum_{i=3}^4 p_i)$$

IPS Location - 101

Process.



Reduced Process.



Equations.

c_1 = same as 100 location

c_3 = same as 001 location

$$c_4 = \frac{x_4 \sum_{24} A_i + \sum_{24} B_i}{x_{g4}}$$

where $\sum_{24} A_i = c_1(1+r_2) + c_3(1+r_2) + \sum_{24} m_i(1+t_i) + \sum_{24} k_i$

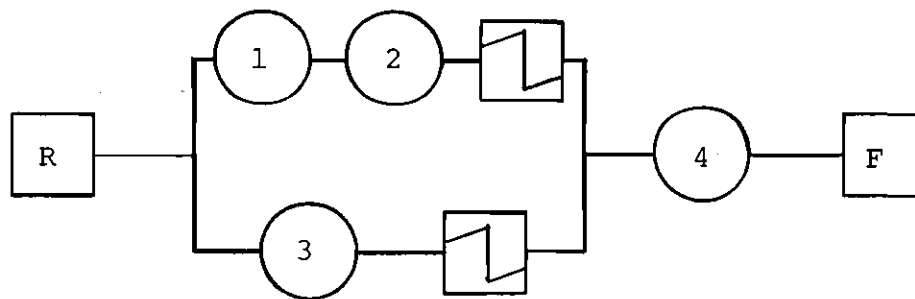
$$X_4 = \left[\text{Min}(K_i, b_i) \right]_{24} \left(1 - \sum_{24} d_i \right)$$

$$\sum_{24} B_i = \sum_{24} (G_i + f_i + s_i)$$

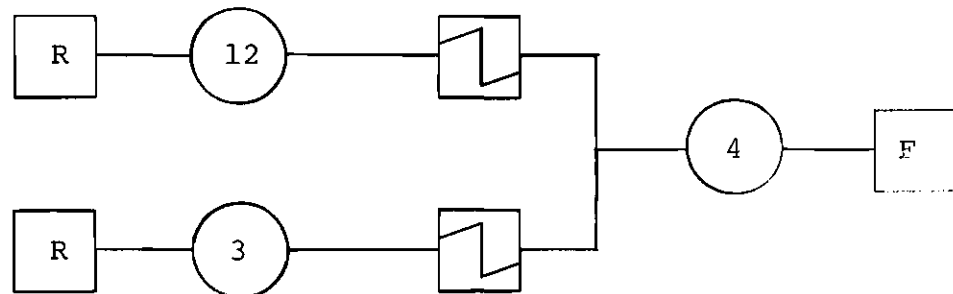
$$X_{g4} = \left[\text{Min}(K_i, b_i) \right]_{24} \left(1 - \sum_{24} d_i \right) \left(1 - \sum_{24} p_i \right)$$

IPS Location - 011

Process.



Reduced Process.



Equations.

c_2 = same as 010 location

c_3 = same as 001 location

$$c_4 = \frac{A_4 X_4 + B_4}{X_{g4}}$$

$$\text{where } A_4 = (c_2 + c_3)(1 + r_4) + m_4(1 + t_4) + k_4$$

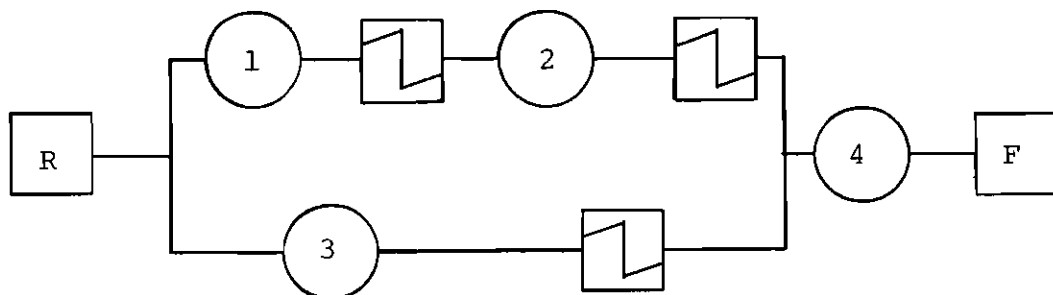
$$X_4 = K_4 b_4 (1 - d_4)$$

$$B_4 = G_4 + f_4 + s_4$$

$$X_{g4} = K_4 b_4 (1 - d_4) (1 - p_4)$$

IPS Location - 111

Process.



Reduced Process. No reduction is possible.

Equations.

c_1 = same as 100 location

c_2 = same as 110 location

c_3 = same as 001 location

$$c_4 = \frac{A_4 X_4 + B_4}{X_{g4}}$$

$$\text{where } A_4 = (c_2 + c_3)(1 + r_4) + m_4(1 + t_4) + k_4$$

$$X_4 = K_4 b_4 (1 - d_4)$$

$$B_4 = G_4 + f_4 + s_4$$

$$X_{g4} = K_4 b_4 (1 - d_4) (1 - p_4)$$

Application of Systems Models

To show the application of unit cost models to different types of systems, a set of five examples has been prepared. The examples are presented in the Appendix. Each example has been discussed in detail before the formulas are applied. A study of the examples will show exactly how the models are used as well as the variety of production systems to which unit cost models can be applied.

Most real systems are not even as complex as these examples. The real processes presented at the beginning of this study, other real processes and textbook examples of processes have been considered without encountering any special problems. In the examples the feedback type system presented the greatest model formulation difficulties.

Real systems are not deterministic, but unit costs can not be treated in accounting procedures as random variables. The probabilistic operation of the process always presents problems in parameter evaluation. Some parameters such as p_i , r_i and t_i can be determined from inspection records in the quality control department. Others, such as d_i , have to

be determined by installing automatic counters on the process equipment. For those operations paid by piece work such counters are usually already installed, or else time cards or "line logs" are available since operations are paid daywork when the equipment is down.

However, some unit costs are subject to important random process variables, and separate studies (usually simulation of the process by computer) should be made to determine costs. One such process is spinning in textile spinning (or throwing) mills. The output in pounds from a spinning operation depends upon a distribution of bobbin weights. The occurrence of breaks in the yarn is a major random variable since the bobbin must be doffed whenever a yarn break occurs.

In this study, modeling forms for sub-system configurations, such as single operations and certain types of sequences, are presented in the belief that all complete systems are composed of combinations of sub-systems in just a few configurations. In order to state that "*all* systems can be modeled," it might be necessary to group production systems into a set of clearly defined, mutually exclusive and exhaustive classes with model building roles for each class. Another approach to a complete classification might be to list all possible process types. Such a classification seems considerably more cumbersome than the method used in this study.

Summary

The canonical form for a production process is shown, and cost models are proposed for the reduced form rather than the original process form. The use of models for canonical process forms is illustrated by an application of cost models to answer the question of in-process storage location. Applications of the methods of this study to five different process types are discussed. The complete applications are shown in the Appendix.

CHAPTER X

CONCLUSIONS AND RECOMMENDATIONS

The initial objectives of this study have each been approached with varying degrees of success. A review of the study at this point makes some definite conclusions possible and also reveals a number of questions posed in the study but not completely answered. The conclusions are listed below. The additional questions are referred to as recommendations.

Conclusions

1. A model has been formulated for product unit cost. It has been shown to have many possible applications to manufacturing processes.
2. The parameters in the model represent values normally determined for cost accounting and process control uses.
3. A standard form has been developed for the model based upon a reduction of all operations between inventory points to a single operation.

Recommendations

1. Further study of standard forms is needed to produce a transformation from the process diagram to the

standard form. A calculating algorithm for the standard form would also be a valuable development.

2. For sequences of dependent operations with more than one practical operating case, further study can be made to determine the application of a model for unit cost. The occurrence of a change in cases would be a random variable.

3. Some process types are easily fitted to the model, but others are more difficult to fit. Further study of processes, such as feedback type processes, would be desirable.

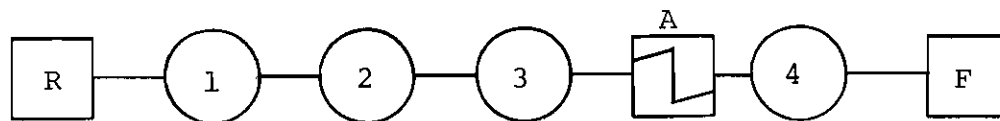
4. A measured work model has been developed only for the single operation case. The extension of measured work models to complex systems can be made. This means that studies of sums of effort levels will have to be considered in sequences of operations.

APPENDIX

EXAMPLES OF COST MODEL APPLICATIONS

Series Sequence of Operations

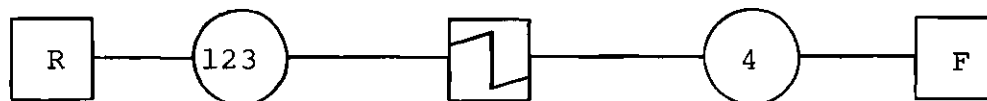
The process configuration is shown below.



We assume an IPS unit after operation 3. The process parameters are shown on page 110.

Process Description - Raw material from storage R goes to operation 1, the output from 1 is the input for 2, the output from 2 is input to 3, and the output from 3 goes to IPS unit A. When drawn from A into operation 4, the output goes to finished goods inventory F. There are two independent sequences in the process. For both, the material balance is normal one to one. Downtime can occur at all four operations. Raw material from storage R is inspected before operation 1 with resulting reject proportion r_1 , and material from IPS unit A is inspected before entering operation 4 with resulting reject proportion r_4 . There is an inspection after operation 3 with resulting scrap proportion p_3 . Purchased material enters all four operations at unit costs m_1 , m_2 , m_3 and m_4 . These purchased materials are overused to proportions t_1 , t_2 , t_3 and t_4 .

Reduced Form - For the cost calculation the process is considered in most reduced form as shown below.



Computed Costs - Unit costs are computed for the outputs of operations 3 and 4. Unit costs are not calculated after operations 1 and 2 because there would be no identifiable product at these points.

Equations:

$$c_3 = \frac{x_3 \sum_{i=1}^3 A_i + \sum_{i=1}^3 B_i}{x_{g3}}$$

$$\text{where } \sum_{i=1}^3 A_i = c_0(1+r_1) + \sum_{i=1}^3 m_i(1+t_i) + \sum_{i=1}^3 k_i$$

$$x_3 = [\text{Min}(K_i, b_i)] (1 - \sum_{i=1}^3 d_i)$$

$$\sum_{i=1}^3 B_i = \sum_{i=1}^3 (G_i + f_i + s_i)$$

$$x_{g3} = [\text{Min}(K_i, b_i)] (1 - \sum_{i=1}^3 d_i) (1 - \sum_{i=1}^3 p_i)$$

$$c_4 = \frac{A_4 X_4 + B_4}{X_{g4}}$$

$$\text{where } A_4 = c_3(1+r_4) + m_4(1+t_4) + k_4$$

$$X_4 = K_4 b_4 (1-d_4)$$

$$B_4 = G_4 + f_4 + s_4$$

$$X_{g4} = K_4 b_4 (1-d_4) (1-p_4)$$

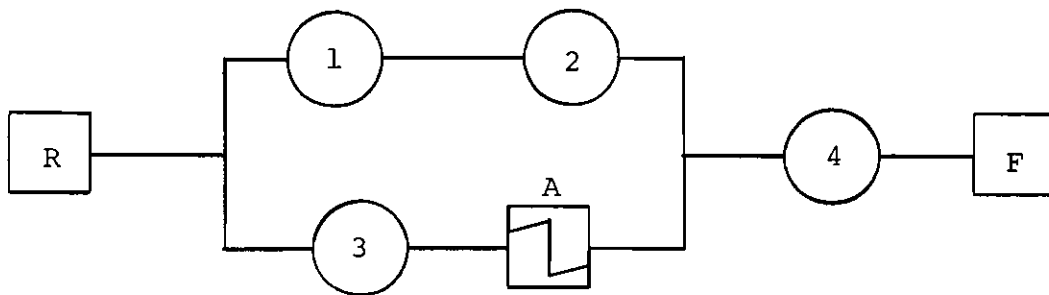
Unit Cost Calculations:

$$c_3 = \$106.21/\text{unit}$$

$$c_4 = \$124.13/\text{unit}$$

Series-Parallel Operations

The process configuration is shown below.



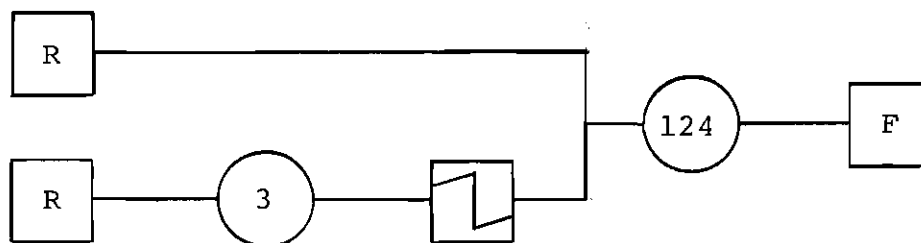
An IPS unit, A, is assumed for this example after operation 3.

Process Description - Raw material from storage R goes simultaneously to operations 1 and 3. Operation 3 output goes to IPS unit A. Thus, operation 3 is independent of the

remainder of the process. Input raw material to operation 3 is inspected and a proportion, r_3 , is rejected. Input purchased material at unit cost m_3 with overusage proportion t_3 enters operation 3. The downtime proportion is d_3 , and inspection of the output of operation 3 results in scrap proportion p_3 . The material balance for the entire process is normal and one to one.

Raw material entering operation 1 after an inspection with fraction defective r_1 , is combined with purchased material, and the output from operation 1 goes to operation 2 as input without further inspection. The output from 2 goes directly to operation 4. Also entering operation 4 is worked material from IPS unit A, but this material is inspected resulting in fraction defective r_4 . These two inputs combine with purchased material m_4 to form the total input to this operation. The output is inspected and scrap proportion p_3 is lost before the output enters the storage F for finished units. All process parameters are shown on page 110.

Reduced Form - This cost calculation is made for the reduced form shown below.



The computed costs are c_3 and c_4 . The inputs to the sequence 1, 2 and 4 are all known except c_3 , so it would have to be determined before cost c_4 .

Equations:

$$c_3 = \frac{A_3 X_3 + B_3}{X_{g3}}$$

$$\text{where } A_3 = c_0(1+r_3) + m_3(1+t_3) + k_3$$

$$X_3 = K_3 b_3 (1-d_3)$$

$$B_3 = G_3 + f_3 + s_3$$

$$X_{g3} = K_3 b_3 (1-d_3)(1-p_3)$$

$$c_4 = \frac{X_4 \sum_{124} A_i + \sum_{124} B_i}{X_{g4}}$$

$$\text{where } \sum_{124} A_i = c_0(1+r_1) + c_3(1+r_4) + \sum_{124} m_i(1+t_i) + \sum_{124} k_i$$

$$X_4 = [\text{Min}(K_i b_i)] (1 - \sum_{124} d_i)$$

$$\sum_{124} B_i = \sum_{124} (G_i + f_i + s_i)$$

$$X_{g4} = [\text{Min}(K_i b_i)] (1 - \sum_{124} d_i) (1 - \sum_{124} p_i)$$

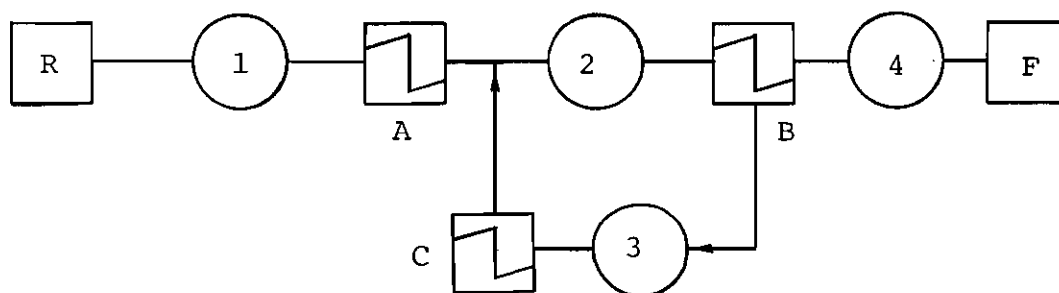
Unit Cost Calculations:

$$c_3 = \$ 66.58/\text{unit}$$

$$c_4 = \$123.26/\text{unit}$$

A Feedback System

The process configuration is shown below.



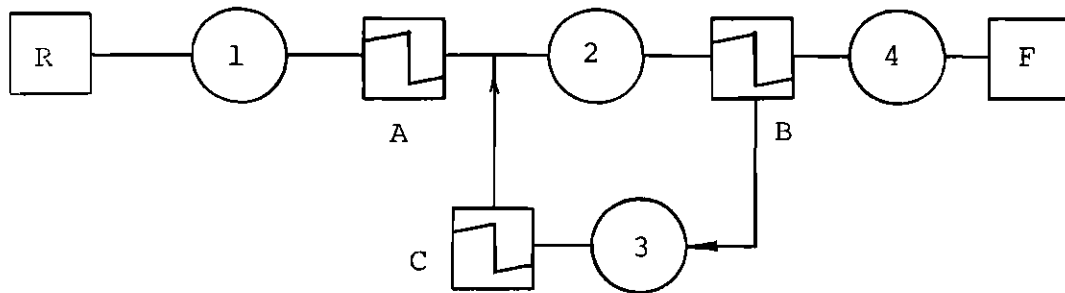
Assume that IPS units are installed after operations 1, 2 and 3. Also assume a normal one to one material balance.

Process Description - The flow of material through the process presents the distinguishing feature of this production system. Units from storage R go through operation 1 to storage A. For each unit taken from A for operation 2, one unit is also taken from storage C. After operation 2 and storage B each processed unit goes both to operations 4 and 3. This might occur in a process if a container or fixture, or a part required during operation 2 but not an integral part of the unit, is prepared in operation 3, used in operation 2 and separated from the output unit at storage B. Operation 4 is the final operation and its output goes to storage F. To simplify the description of this process, IPS units are

installed at all possible locations. Unit costs c_1 , c_3 and c_4 can be determined as in all previous processes, but operation 2 does not have to pay for all worked material from operation 3. The problem for this type feedback process is to determine the fraction of material from operation 3 that is used in operation 2 and is charged to operation 2.

The loss to each unit in operation 2 is $r_2 + p_2$. Material going back to operation 3 is inspected with loss r_3 . The total fraction is $[r_2 + p_2 + r_3]$. This becomes the multiplier for c_3 in the unit cost for operation 2. The process parameters are shown on page 110.

Reduced Form - With all possible IPS units utilized, four independent single operations result.



The computed costs are c_1 , c_2 , c_3 and c_4 .

$$c_1 = \frac{A_1 X_1 + B_1}{X_{g1}}$$

$$\text{where } A_1 = c_0(1+r_1) + m_1(1+t_1) + k_1$$

$$X_1 = K_1 b_1 (1-d_1)$$

$$B_1 = G_1 + f_1 + s_1$$

$$X_{g1} = K_1 b_1 (1-d_1) (1-p_1)$$

$$c_2 = \frac{A_2^* X_2 + B_2}{X_{g2}}$$

$$\text{where } A_2^* = c_3 [r_3 + r_2 + p_2] + c_1 (1+r_2) + m_2 (1+t_2) + k_2$$

$$X_2 = K_2 b_2 (1-d_2)$$

$$B_2 = G_2 + f_2 + s_2$$

$$X_{g2} = K_2 b_2 (1-d_2) (1-p_2)$$

$$c_3 = \frac{A_3 X_3 + B_3}{X_{g3}}$$

$$\text{where } A_3 = c_0 (1+r_3) + m_3 (1+t_3) + k_3$$

$$X_3 = K_3 b_3 (1-d_3)$$

$$B_3 = G_3 + f_3 + s_3$$

$$X_{g3} = K_3 b_3 (1-d_3) (1-p_3)$$

$$c_4 = \frac{A_4 X_4 + B_4}{X_{g4}}$$

$$\text{where } A_4 = c_2(1+r_4) + m_4(1+t_4) + k_4$$

$$X_4 = K_4 b_4 (1-d_4)$$

$$B_4 = G_4 + f_4 + s_4$$

$$X_{g4} = K_4 b_4 (1-d_4) (1-p_4)$$

Unit Cost Calculations:

$$c_1 = \$ 5.65/\text{unit}$$

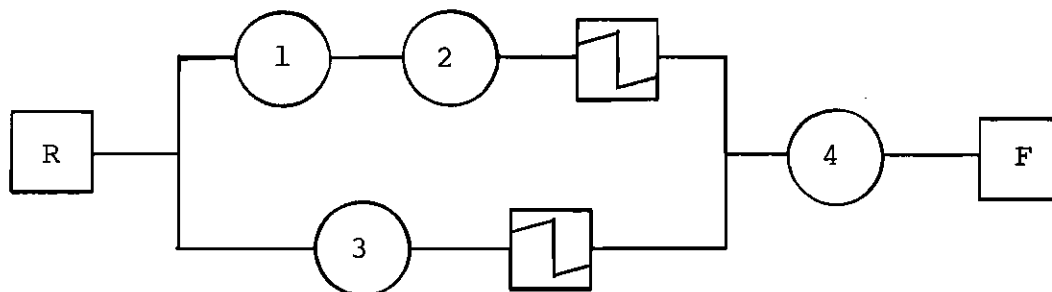
$$c_2 = \$36.66/\text{unit}$$

$$c_3 = \$66.58/\text{unit}$$

$$c_4 = \$52.08/\text{unit}$$

A Regenerative System

The process configuration is shown below.

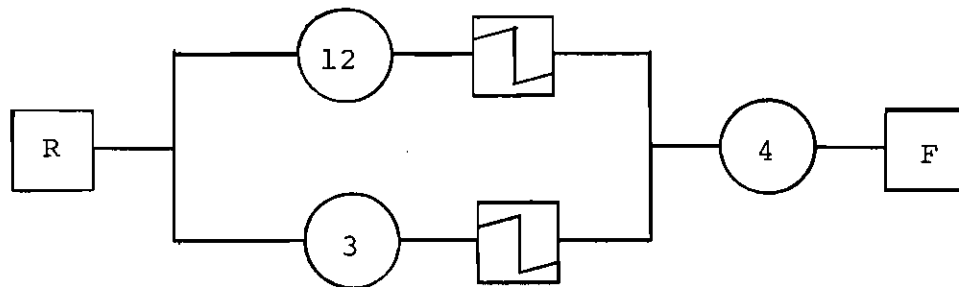


Assume IPS units after operations 2 and 3. The process parameters are given on page 110.

Process Description - Except for a gradual linear decrease in the efficiency of operation 3, this process is identical to the one on page 130. The capacity of operation

3 is considered fixed, and to show the decrease in operation efficiency the capacity factor, b_3 , is assumed to start at a maximum of 0.90 and then decreased linearly at a rate of 0.05 per day. After 35 days of operation b_3 has a value of 0.20 and the operation is "regenerated." Then on the 36th day it starts with b_3 at 0.90. For the purpose of the calculations shown, it is further assumed that b_3 decreases in steps of 0.05 rather than continuously, as would be expected in practice. The material balance is again normal and one to one for all four operations.

Reduced Form - In the reduced form the process would be as shown below.



The computed costs are c_2 , c_3 and c_4 .

Equations:

$$c_2 = \frac{x_2 \sum_{i=1}^2 A_i + \sum_{i=1}^2 B_i}{x_{g2}}$$

$$\text{where } \sum_{i=1}^2 A_i = c_0(1+r_1) + \sum_{i=1}^2 m_i(1+t_i) + \sum_{i=1}^2 k_i$$

$$x_2 = [\text{Min}(K_i b_i)_{1,2}] (1 - \sum_{i=1}^2 d_i)$$

$$\sum_{i=1}^2 B_i = \sum_{i=1}^2 (G_i + f_i + s_i)$$

$$x_{g2} = [\text{Min}(K_i b_i)_{1,2}] (1 - \sum_{i=1}^2 d_i) (1 - \sum_{i=1}^2 p_i)$$

$$c_3 = \frac{A_3 x_3 + B_3}{x_{g3}}$$

$$\text{where } A_3 = c_0(1+r_3) + m_3(1+t_3) + k_3$$

$$x_3 = K_3 b_3 (1-d_3)$$

$$B_3 = G_3 + f_3 + s_3$$

$$x_{g3} = K_3 b_3 (1-d_3) (1-p_3)$$

$$c_4 = \frac{A_4 x_4 + B_4}{x_{g4}}$$

$$\text{where } A_4 = (c_2 + c_3)(1+r_4) + m_4(1+t_4) + k_4$$

$$x_4 = K_4 b_4 (1-d_4)$$

$$B_4 = G_4 + f_4 + s_4$$

$$x_{g4} = K_4 b_4 (1-d_4) (1-p_4)$$

Costs for "Regenerated" Operations - The cost equation given on page 138 for operation 3 depends upon a fixed b_3 parameter; but for a regenerated operation there is also the question of how unit cost changes as a function of the regeneration interval. The cost of regeneration is an expense. Suppose this cost is \$3500 or \$100 per day on a 35-day regenerating cycle. If we call this cost per day α ;

$$\alpha = \frac{\text{cost in dollars}}{\text{cycle in days}} = \frac{c}{d}$$

the overhead elements for operation 3 (which is the only operation involved) is;

$$c_3^0 = \frac{1}{1 - p_3} \left(\frac{f_3 + s_3 + \frac{c}{d}}{k_3 b_3 (1 - d_3)} \right)$$

This means that regenerating cost, as an additional overhead cost, is separated from the other expense costs for evaluation. In the expression above, b_3 is also determined by d and is;

$$b_3 = 0.90 - 0.05d$$

so that;

$$c_3^0 = \frac{1}{1 - p_3} \left(\frac{f_3 + s_3 + \frac{c}{d}}{K_3 (.90 - .05d) (1 - d_3)} \right)$$

and taking the derivative of c_3^0 with respect to d , we have;

$$\frac{\partial}{\partial d} [c_3^0] = \frac{\partial}{\partial d} \left[\frac{1}{1 - p_3} \left(\frac{c/d}{K_3 (.90 - .05d) (1 - d_3)} \right) \right]$$

$$\frac{\partial}{\partial d} \left(\frac{c}{[.90(1 - p_3)K_3(1 - d_3)]d - [.05(1 - p_3)K_3(1 - d_3)]d^2} \right)$$

$$= \frac{-c(k_1^* - 2k_2^*d)}{[k_1^*d - k_2^*d^2]^2}$$

where

$$k_1^* = .90(1 - p_3)K_3(1 - d_3)$$

$$k_2^* = .05(1 - p_3)K_3(1 - d_3)$$

Setting this derivative equal to zero and solving for d_0 , we have;

$$d_0 = \frac{k_1^*}{2k_2^*}$$

The calculated values are shown below using the parameters given in Table 9. The minimum c_3^0 value occurs for $d = 8.91$ days. At 9 days the α value is \$389 and results in a minimum unit expense cost.

Then using a regenerating cycle of 9 days, we determine the average b_3 value over the 9-day interval in order to determine the expense unit cost for operation 3. Since the c_3 cost is adjusted in this manner we can write it as c_3^* .

Unit Cost Calculations:

$$c_2 = \$31.26$$

For c_3^* ;

$$k_1^* = 677$$

$$K_2^* = 37.6 \approx 38$$

$$d_0 = \frac{677}{76} = 8.91 \text{ days; use } d_0 = 9 \text{ days}$$

$$\alpha = \frac{\$3500}{9} = \$389/\text{day}$$

$$c_3^0 = \$2.77/\text{unit}$$

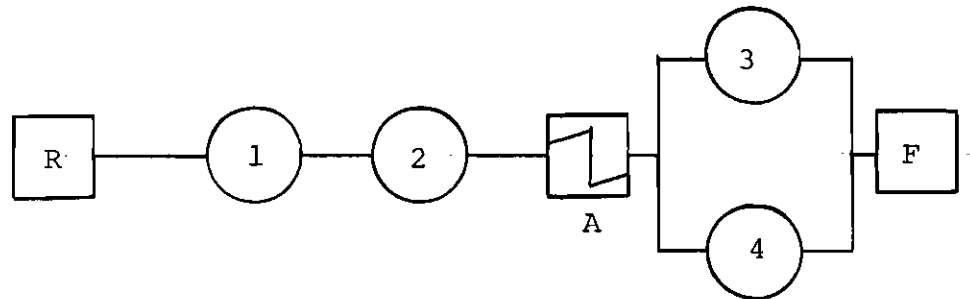
$$c_3^* = \$68.53/\text{unit}$$

and for c_4 ;

$$c_4 = \$118.46/\text{unit}$$

A Batch Process

The process configuration is shown below.

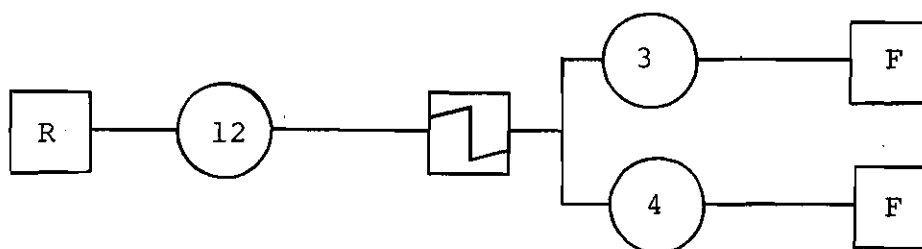


Assume as IPS unit after operation 2. The parameters for the process are given on page 110.

Process Description - A batch process contains identical parallel operations and is, therefore, a special case of the series parallel process discussed in Series-Parallel Operations. It presents no special difficulties that do not apply to all series parallel models.

In this case the storage R provides the input to operation 1 which is in series with operation 2. An IPS unit after operation 2 absorbs the output of operation 2. Both operation 3 and 4 draw input units from IPS unit A, and the output from these batch operations goes to storage F after final inspection. As far as unit cost computations are involved, it makes no difference whether the operation is continuous or a batch process.

Reduced Form - The reduced form used for unit cost computation is shown on page 143.



The computed costs are c_2 , c_3 and c_4 .

Equations:

$$c_2 = \frac{X_2 \sum_{i=1}^2 A_i + \sum_{i=1}^2 B_i}{X_{g2}}$$

$$\text{where } \sum_{i=1}^2 A_i = c_0(1+r_1) + \sum_{i=1}^2 m_i(1+t_i) + \sum_{i=1}^2 k_i$$

$$X_2 = [\text{Min}(K_i b_i)_{1,2}] (1 - \sum_{i=1}^2 d_i)$$

$$\sum_{i=1}^2 B_i = \sum_{i=1}^2 (G_i + f_i + s_i)$$

$$X_{g2} = [\text{Min}(K_i b_i)_{1,2}] (1 - \sum_{i=1}^2 d_i) (1 - \sum_{i=1}^2 p_i)$$

$$c_3 = \frac{A_3 X_3 + B_3}{X_{g3}}$$

$$\text{where } A_3 = c_2(1+r_3) + m_3(1+t_3) + k_3$$

$$X_3 = K_3 b_3 (1-d_3)$$

$$B_3 = G_3 + f_3 + s_3$$

$$X_{g3} = K_3 b_3 (1-d_3) (1-p_3)$$

$$c_4 = \frac{A_4 X_4 + B_4}{X_{g4}}$$

$$\text{where } A_4 = c_2 (1+r_4) + m_4 (1+t_4) + k_4$$

$$X_4 = K_4 b_4 (1-d_4)$$

$$B_4 = G_4 + f_4 + s_4$$

$$X_{g4} = K_4 b_4 (1-d_4) (1-p_4)$$

Unit Cost Calculations:

$$c_2 = \$ 31.26$$

$$c_3 = \$100.43$$

$$c_4 = \$ 46.41$$

A MODEL UNIT COST AND AN ACTUAL UNIT COST

This comparison is presented to illustrate two points. First, that a cost determined using the model does accurately determine unit product cost; and, second, that the model parameters represent values presently used in costing and, therefore, the model can be easily applied.

A process with a unit cost of \$947.13 per thousand units is to be evaluated by the model. A diagram of the process is presented as Figure 16 showing all parameters for the unit cost model. The values shown in circles are the number of thousands of parts required to produce 1000 good units of the final output product. The questions are (1) whether or not a unit cost computed by the model is equal to the actual cost; and (2) how is the model applied to an established cost? The cost model is;

$$c_1 = \frac{A_1 X_1 + B_1}{X_{g1}}$$

where c_1 = cost per 1000 units

$$A_1 = \sum c_{0j}(1+r_{1j}) + \sum m_{1k}(1+t_{1k})$$

$$X_1 = K_1 b_1 (1-d_1) = 1159 \text{ units per day}$$

$$B_1 = G_1 + f_1 + s_1 = G_1 + [f_1 + s_1] = \$369.92 + \$310.48$$

$$X_{g1} = 1159 \text{ units per day}$$

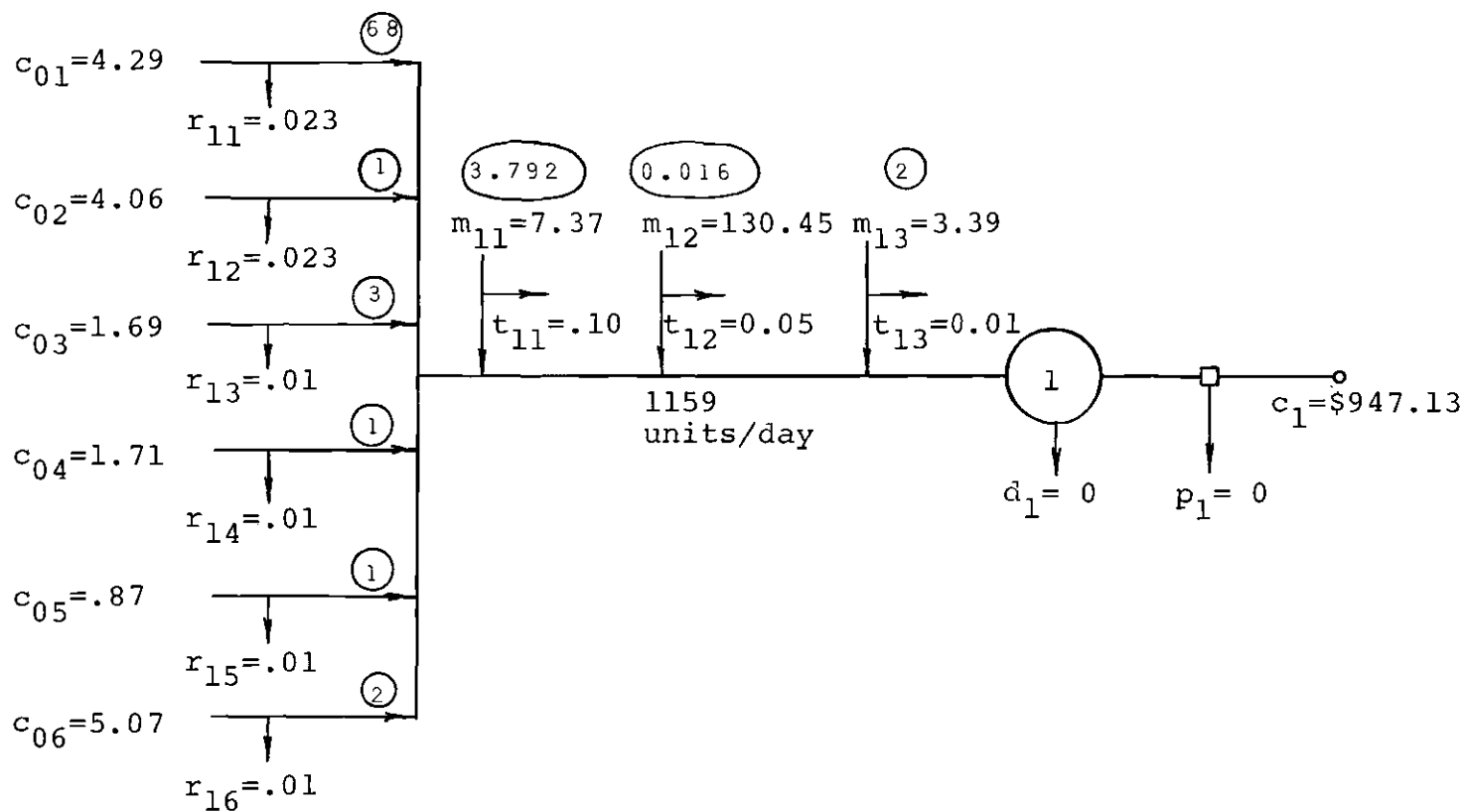


Figure 16. Diagram of a Single Operation Showing the Single Operation Model Parameters. Costs are in Dollars per Thousand Units

Notice that instead of $f_1 + s_1$, an "expense rate" is used in this case of \$0.84 per dollar of direct labor. Also, the input and output for the operation is the same value. This is an "experience-with-the-process" estimate. The parameters k_1 , b_1 , d_1 and p_1 are not measured. To apply the model $d_1 = 0$, $p_1 = 0$, $b_1 = 1.00$ and $K_1 = 1159$.

There are six different parts taken from previous operations in the same plant and three different purchased items. The parameter values for the unit cost model are shown in Table 11, and the term A_1 , given on page 145, can be expanded using these parameter values as follows:

$$\begin{aligned} A_1 = & 68c_{01}(1+r_{11}) + c_{02}(1+r_{12}) + 3c_{03}(1+r_{13}) \\ & + c_{04}(1+r_{14}) + c_{05}(1+r_{15}) + 2c_{06}(1+r_{16}) \\ & + 3.792m_{11}(1+t_{11}) + 0.016m_{12}(1+t_{12}) \\ & + 2m_{13}(1+t_{13}) \end{aligned}$$

After substituting parameter values into this expression,

$A_1 = \$360.34$. The cost is then;

$$c_1 = \frac{360.34(1.159) + 369.69 + 310.48}{1.159} = \$947.20$$

Table 11. Parameter Values for Model Cost to Actual Cost Comparison

System Parameter	Symbol	Units	Operation 1
Operation Capacity	K_1	units/shift	1159
Capacity Factor	b_1	-	1.00
Scrap Loss Factor	p_1	-	0
Downtime Factor	d_1	-	0
Worked Item	c_{01}	\$/1000 units	4.29
Reject Factor	r_{11}	-	0.23
Worked Item	c_{02}	\$/1000 units	4.06
Reject Factor	r_{12}	-	0.23
Worked Item	c_{03}	\$/1000 units	1.69
Reject Factor	r_{13}	-	.01
Worked Item	c_{04}	\$/1000 units	1.71
Reject Factor	r_{14}	-	.01
Worked Item	c_{05}	\$/1000 units	.87
Reject Factor	r_{15}	-	.01
Worked Item	c_{06}	\$/1000 units	5.07
Reject Factor	r_{16}	-	.01
Purchased Item	m_{11}	\$/1000 yards	7.37
Reject Factor	t_{11}	-	.10
Purchased Item	m_{12}	\$/1000 pounds	130.45
Reject Factor	t_{12}	-	.05
Purchased Item	m_{13}	\$/1000 units	3.39
Reject Factor	t_{13}	-	.01
Total Daywork Labor	G_1	\$/shift	369.62
Total Variable Overhead	k_1	-	0
Total Fixed and Semifixed Overhead	$f_1 + s_1$	\$/shift	310.48
Calculated Unit Cost	c_1	\$/unit	947.20

Thus, unit cost by the model is equal to the actual unit cost. It is clear in making the comparison that the use of the model may require more detailed process data, for example p and d . This cost was written by the author in 1956. At that time the p_i value was reported for this production process in a monthly report by the inspection department, and the d_i value could have been estimated from the production line time log used by the payroll department.

It is also clear that this comparison does not prove the validity of the model. It only shows an application to a process with an established cost.

GLOSSARY OF SYMBOLS

A	Material, labor and overhead elements applied as unit costs.
B	Material, labor and overhead elements applied as total dollars.
b_i	A capacity factor which is applied to the operation factor K_i so that $K_i b_i$ represents the actual expected output from an operation. It is a dimensionless number.
br_i	The base rate for operation i .
c_i	Unit cost of the product after operation i .
c_{i-1}	Unit cost of worked material from operation $i-1$.
$c_{i-1,j}$	Unit cost of the j 'th worked material from operation $i-1$.
c_i^M	Material cost portion of unit cost c_i .
c_i^L	Labor cost portion of unit cost c_i .
c_i^O	Overhead cost portion of unit cost c_i .
d_i	Downtime loss shown in the model as a proportional adjustment to K_i .
Ef_i	The performance level expected at operation i .
f_i	Fixed portion of overhead cost expressed in dollars of total cost.
G_i	Guaranteed day rate cost that applies to operations on daywork.

IPS	An abbreviation for In-Process Storage Unit.
K_i	The capacity for operation i .
k_i	Overhead costs per unit.
m_i	Purchased material cost per unit.
m_{ij}	Unit cost of the j 'th purchased material used at operation i .
p_i	Fraction loss in finished units.
r_i	Reject proportion or fraction applied to incoming material.
r_{ij}	Fraction rejected for the j 'th input worked material used at operation i .
s_i	Semifixed portion of overhead cost expressed in total dollars assigned to operation i .
sm_i	Standard minutes per unit for operation i .
t_i	A reject factor for purchased material representing the fraction of the purchased material rejected at the operation.
t_{ij}	Fraction rejected of the j 'th purchased material used at operation i .
X_i	Input quantity to operation i in units.
X_{gi}	Output from operation i in good units.

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VITA

Douglas Hynds Hutchinson, the third son of Patrick Hynds and Anna (née Oakley) Hutchinson, was born on April 9, 1932 in New York City. During his first nine years the family lived in New York City; Connecticut; Detroit, Michigan; and Mt. Lebanon, Pennsylvania. In 1946 the family moved to Atlanta, Georgia, and he attended Boys High School and Henry Grady High School, graduating from the latter in 1949.

In the fall of 1949 Mr. Hutchinson entered the Georgia Institute of Technology (following his two older brothers) and graduated in 1953 with the degree Bachelor of Industrial Engineering. As an undergraduate student he was an officer of Alpha Pi Mu, Industrial Engineering Honor Society, and a member of the Phi Kappa Sigma social fraternity. During the summer quarters as an undergraduate student he worked for the Lockheed Aircraft Corporation in Marietta, Georgia, and the Everett Waddy Company in Richmond, Virginia.

After graduation, Mr. Hutchinson was employed as an industrial engineer by the National Carbon Company, A Division of Union Carbide and Carbon Corporation, but after only two months was drafted into the Army Signal Corps. He took basic training at Fort Jackson, South Carolina, and was trained as a high speed radio operator at Camp Gordon, Georgia. In July 1954 he sailed to Bremerhaven, Germany, and was stationed in

Boblingen, Germany, near Stuttgart. As the team chief of a mobile radio unit with the 97th Signal Operations Battalion he traveled to many locations in southern Germany, and during the summer of 1955 was attached to the II French Corps in Koblenz, Germany. He was separated from active duty at Camp Kilmer, New Jersey, in August 1955 and honorably discharged from the Army Signal Corps in August 1961 with the enlisted rank of corporal.

Returning to Union Carbide Corporation in Charlotte, North Carolina, in 1955, Mr. Hutchinson performed a variety of assignments as an industrial engineer. He estimated unit costs for competitive bids on government contracts, designed and installed production systems for the awarded contracts, and performed the usual industrial engineering functions in wage administration and process design. He determined the 1957 normal cost system for the plant.

In 1958, obtaining a one year leave of absence from his company, he returned to Georgia Tech to complete his work and residence requirements toward the degree Master of Science in Industrial Engineering. He completed his thesis, *In-Process Storage for Continuous Production Lines*, during the next four years while again with Union Carbide Corporation. The degree was awarded in June 1962.

In April 1959 he was made the statistician for the Consumer Products Division of the corporation and was transferred to Cleveland, Ohio. In this position he traveled

extensively to company plants in Iowa, Ohio, Vermont and North Carolina performing quality control and statistical process analysis functions. With Dr. J.N. Berrettoni of Western Reserve University he wrote a statistical training manual for the company and assisted Dr. Berrettoni in the teaching of this course to management and engineering personnel. He established company procedures for statistical de-bugging of process machinery and wrote a manual for the company formalizing these procedures. Mr. Hutchinson was a lecturer in statistics for the evening division of Western Reserve University during the 1959-1960 and 1960-1961 school years.

On June 10, 1961 he married Doreen Alicia Grant in Charlotte, North Carolina. They have a son, David Christopher.

In 1962 he joined the faculty of the School of Industrial Engineering of the Georgia Institute of Technology. While teaching as an instructor he enrolled again in the graduate division and studied for the Doctor of Philosophy degree in industrial engineering.

During the summer of 1964 Mr. Hutchinson was employed by the International Paper Company in Mobile, Alabama, as a statistical consultant. While in Mobile he took flying instructions and is a licensed private pilot. In the summer of 1966 he was employed by the Burlington Industries Corporation in Greensboro, North Carolina, and he wrote for them a statistical training manual. At the end of the summer he presented

a one week short course for industrial engineers using the manual as a text. He also participated in short course presentation for the adult extension division at Georgia Tech in statistics, quality control and operations research. He was employed part time during 1966 by the electronics branch of the Georgia Tech Engineering Experiment Station and is a co-author of the report, *Statistical Description of Near Zone Spurious Emissions*.

Mr. Hutchinson is a member of the American Statistical Association, the Society for Industrial and Applied Mathematics, and the Operations Research Society of America. In May 1965 he spoke at a meeting to the Atlanta Chapter of the American Statistical Association on *Some Elementary Stochastic Processes*.